Chapter 10: Introduction to Time Series Modeling and Forecasting B

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Intro to Time Series Modeling & Forecasting

Constructing Time Series Models

The time series model is $y_t=E(y_t)+R_t$ where $E(y_t)=\beta_0+\beta_1x_1+\ldots+\beta_kx_k$ and R_t represents the random residual.

We assume that the residual R_t has mean 0 and constant variance σ^2, but that residuals are autocorrelated, $Cor(R_t,R_{t+1})\neq 0$.

The time series model consists of a pair of (sub) models,
one for the deterministic component $E(y_t)$ and
one for the autocorrelated residuals R_t.

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Constructing Time Series Models

The deterministic component selected in the same way as for previous regression models, i.e.

$$E(y_t) = \beta_0 + \beta_1 x_t$$

$$E(y_t) = \beta_0 + \beta_1 x_t + \beta_2 x_t^2$$

$$E(y_t) = \beta_0 + \beta_1 x_t + \beta_2 t + \beta_3 x_t t$$

Lagged Independent Variables Model

$$E(y_t) = \beta_0 + \beta_1 x_{t-1}$$

$$E(y_t) = \beta_0 + \beta_1 x_{t-1} + \beta_2 x_{t-1}^2$$

Delayed effect from t-1

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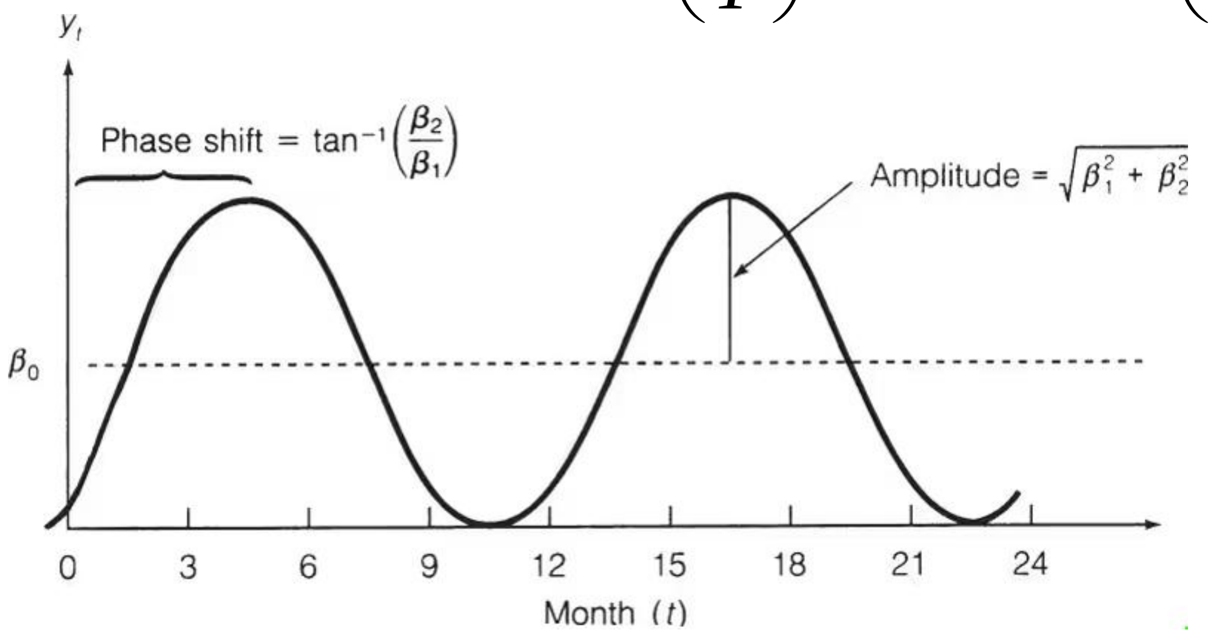
Constructing Time Series Models

Quite often, the deterministic component has distinct seasonal patterns, which can be modeled with

cosines and sines, T=12 months

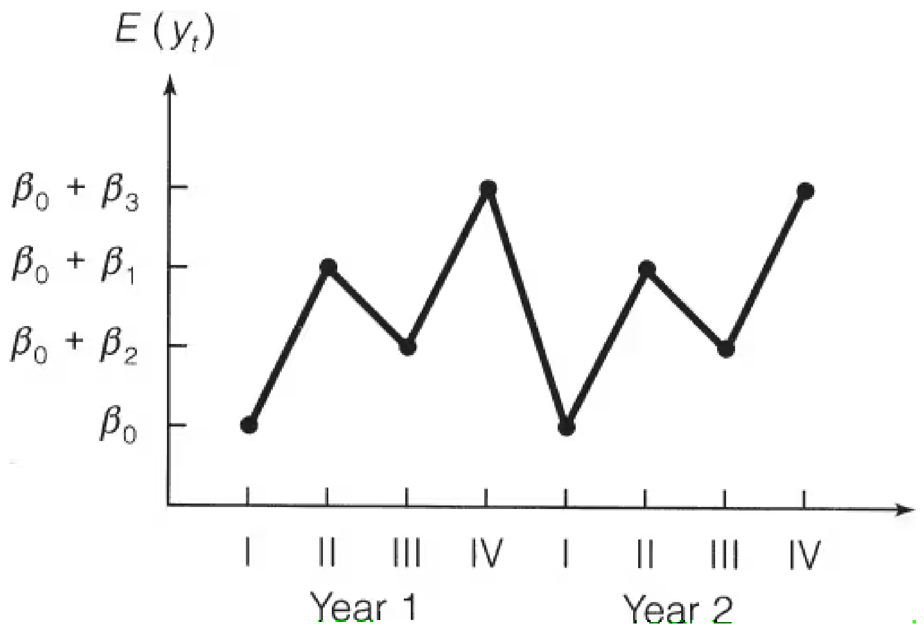
$$E(y_t) = \beta_0 + \beta_1 \cos 2\pi\left(\frac{1}{T}\right)t + \beta_2 \sin 2\pi\left(\frac{1}{T}\right)t$$



$$E(y_t) = \beta_0 + \beta_1 \sin(2\pi t/T + \theta)$$

Annual four seasons

$$E(y_t) = \beta_0 + \beta_1 S_1 + \beta_2 S_2 + \beta_3 S_3$$



$$Q_1 = \begin{cases} 1 \ if \ spring \\ 0 \ if \ not \end{cases}$$

$$Q_2 = \begin{cases} 1 \ if \ summer \\ 0 \ if \ not \end{cases}$$

$$Q_3 = \begin{cases} 1 \ if \ fall \\ 0 \ if \ not \end{cases}$$

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Constructing Time Series Models

Choosing the residual component R_t depends on the pattern of autocorrelation. The general autoregressive model is

$$R_t = \phi_1 R_{t-1} + \phi_2 R_{t-2} + \cdots + \phi_1 R_{t-p} + \varepsilon_t \ , \quad E(\varepsilon_t)=0,\ Var(\varepsilon_t)=\sigma^2,\ Cor(\varepsilon_t,\varepsilon_{t-1})=0$$

and the simplest of which is the first order AR model

$$R_t = \phi R_{t-1} + \varepsilon_t$$

In practice, the autocorrelation generally decreases as time increases.

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Constructing Time Series Models

We can “whiten” or uncorrelate our residuals by “differencing”

$$R_t = \phi R_{t-1} + \varepsilon_t$$

$$y_t = \beta_0 + \beta_1 t + R_t$$

$$y_{t-1} = \beta_0 + \beta_1 (t-1) + R_{t-1}$$

$$\phi y_{t-1} = \phi\beta_0 + \phi\beta_1 (t-1) + \phi R_{t-1}$$

$$y_t - \phi y_{t-1} = \beta_0 - \phi\beta_0 + \beta_1[t - \phi(t-1)] + R_t - \phi R_{t-1}$$

$y_t^* = \beta_0^* + \beta_1^* t^* + \varepsilon_t$, $E(\varepsilon_t)=0$, $Var(\varepsilon_t)=\sigma^2$, $Cor(\varepsilon_t,\varepsilon_{t-1})=0$.

$\hat{\phi} = \left[\sum \varepsilon_t^2 / (n-1)\right] / s_\varepsilon$, $\tilde{\beta}^* = (X^{*\prime} X^*)^{-1} X^{*\prime} y^*$, transform back $\tilde{\beta}_0 = \tilde{\beta}_0^* / (1-\hat{\phi})$ and $\tilde{\beta}_1 = \tilde{\beta}_1^*$

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Fitting Time Series Models with Autoregressive Errors

SALES35.txt

Example: Data on annual sales y for 35 years.
The model is $E(y)=\beta_0+\beta_1 t$.

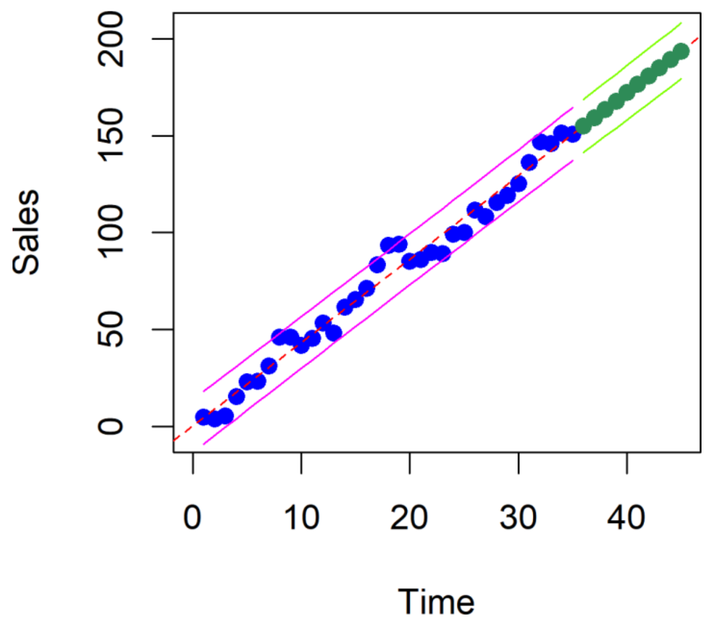
$$\hat{y}=\hat{\beta}_0+\hat{\beta}_1 x_1 = 0.4015+4.2956t$$

```
              Estimate Std. Error    t value     Pr(>|t|)
(Intercept) 0.4015126  2.2057083  0.1820334 8.566701e-01
t           4.2956303  0.1068669 40.1960721 1.306377e-29

Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)
Model      1  65875   65875  1615.7 < 2.2e-16 ***
Residuals 33   1345      41
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

"s,R-squared,adj R-squared"
 6.3852423 0.9799845 0.9793780
```



Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	65875	65875	1615.72	<.0001
Error	33	1345.45355	40.77132		
Corrected Total	34	67221			

Root MSE	6.38524	R-Square	0.9800
Dependent Mean	77.72286	Adj R-Sq	0.9794
Coeff Var	8.21540		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	0.40151	2.20571	0.18	0.8567
T	1	4.29563	0.10687	40.20	<.0001

T	SALES	RESIDUAL
1	4.8	0.102857
2	4	-4.99277
3	5.5	-7.7884
4	15.6	-1.98403
5	23.1	1.220336
6	23.3	-2.87529
7	31.4	0.929076
8	46	11.23345
9	46.1	7.037815
10	41.9	-1.45782
11	45.5	-2.15345
12	53.5	1.550924
13	48.4	-7.84471
14	61.6	1.059664
15	65.6	0.764034
16	71.4	2.268403
17	83.4	9.972773
18	93.6	15.87714
19	94.2	12.18151
20	85.4	-0.91412
21	86.2	-4.40975
22	89.9	-5.00538
23	89.2	-10.001
24	99.1	-4.39664
25	100.3	-7.49227
26	111.7	-0.3879
27	108.2	-8.18353
28	115.5	-5.17916
29	119.2	-5.77479
30	125.2	-4.07042
31	136.3	2.73395
32	146.8	8.938319
33	146.1	3.942689
34	151.4	4.947059
35	150.9	0.151429

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Fitting Time Series Models with Autoregressive Errors

SALES35.txt

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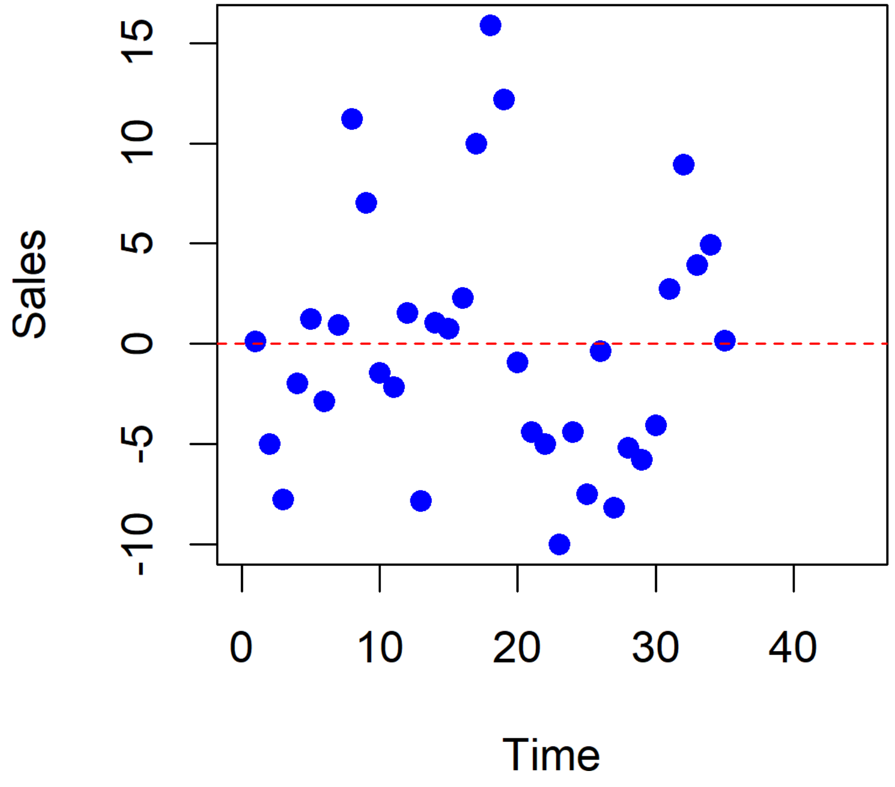
Example: Data on annual sales y for 35 years.
However, the Durbin-Watson statistic d=0.821<d_L=1.40 for α=0.05, n=35, k=1.
This means there is statistically significant AR(1) correlation.

Although the regression estimates are unbiased $E(\hat{\beta}_0) = \beta_0$ and $E(\hat{\beta}_1) = \beta_1$, when we assume no correlation and there is in fact correlation, the the standard errors are usually smaller than the true standard errors and, t-values computed by the methods will usually be inflated and will lead to a higher Type I error rate α.



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Example: Data on annual sales y for 35 years.

To account for autocorrelation, we postulate an AR(1) model,

$y_t = \beta_0 + \beta_1 t + R_t$, $R_t = \phi R_{t-1} + \varepsilon_t$ and estimate β_0, β_1, ϕ by least squares.

Ordinary Least Squares Estimates

SSE	1345.45355	DFE	33
MSE	40.77132	Root MSE	6.38524
SBC	234.156237	AIC	231.045541
MAE	4.85208163	AICC	231.420541
MAPE	13.8946717	HQC	232.119353
Durbin-Watson	0.8207	Regress R-Square	0.9800
		Total R-Square	0.9800

Parameter Estimates

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	0.4015	2.2057	0.18	0.8567
T	1	4.2956	0.1069	40.20	<.0001

Estimates of Autocorrelations

Lag	Covariance	Correlation	-1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
0	38.4415	1.000000	| |********************|
1	22.6661	0.589624	| |************ |

Preliminary MSE	25.0771

Estimates of Autoregressive Parameters

Lag	Coefficient	Standard Error	t Value
1	-0.589624	0.142779	-4.13

Yule-Walker Estimates

SSE	877.685377	DFE	32
MSE	27.42767	Root MSE	5.23714
SBC	223.18683	AIC	218.520786
MAE	4.07427092	AICC	219.29498
MAPE	12.142864	HQC	220.131504
Durbin-Watson	1.8217	Regress R-Square	0.9412
		Total R-Square	0.9869

Parameter Estimates

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	0.4058	3.9970	0.10	0.9198
T	1	4.2959	0.1898	22.63	<.0001

	Least Squares	Autoregressive
R^2	.980	.987
MSE	40.77	27.43
$\hat{\beta}_0$	.4015	.4058
$\hat{\beta}_1$	4.2956	4.2959
Standard error $(\hat{\beta}_0)$	2.2057	3.9970
Standard error $(\hat{\beta}_1)$	.1069	.1898
t statistic for H_0: $\beta_1 = 0$	40.20	22.63
	$(p < .0001)$	$(p < .0001)$
$\hat{\phi}$	—	.5896
t statistic for H_0: $\phi = 0$	—	4.13

$$\hat{y}_t = .4058 + 4.2959t + \hat{R}_t$$
$$\hat{R}_t = 0.5896\hat{R}_{t-1}$$

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Fitting Time Series Models with Autoregressive Errors

```
# read data
mydata <- read.delim("SALES35.txt",header=TRUE,sep="",dec=".")

# parse out variables
n <- nrow(mydata)
t <- c(mydata[,1]) #t time
y <- c(mydata[,2]) #y sales
r <- c(mydata[,3]) #r residual

# fit uncorrelated line model and get statistics
mymodel1 <- lm(y~t)

# get uncorrelated error regression coefficients
b0<-mymodel1$coefficients[1]
b1<-mymodel1$coefficients[2]

# scatter plot with uncorrelated error line
plot(t,y,pch=19,col="blue",xlab='Time',ylab='Sales',xlim=c(0,n))
abline(mymodel1,lty=2,col='red’)

# uncorrelated error residual plot with 0 line
plot(t,r,pch=19,col="blue",xlab='Time',ylab='Sales',xlim=c(0,n))
abline(lm(r~t),r,lty=2,col='red')
```

```
#AR 1 correlation from uncorrelated error model
phihat<-cor(r[1:(n-1)],r[(1+1):n])

# fit correlated line model and get statistics
yast<-y[2:n]-phihat*y[1:n-1]
tast<-t[2:n]-phihat*t[1:n-1]
mymodel2 <- lm(yast~tast)

# get correlated error regression coefficients
b0ast<-mymodel2$coefficients[1]/(1-phihat)
b1ast<-mymodel2$coefficients[2]
bast<-c(b0ast,b1ast)

# fit and intervals
c   <-rep(1,n)      #Ones
X   <-matrix(cbind(c,t),nrow=n,ncol=2) #design matrix
yasthat<-X%*%bast

# scatter plot with uncorrelated error line
plot(t,y,pch=19,col="blue",xlab='Time',ylab='Sales',xlim=c(0,n))
abline(lm(y~t),r,lty=2,col='red')
lines(t,yasthat, type = "l", col = "green")
```

Intro to Time Series Modeling & Forecasting

Forecasting with Time Series Autoregressive Models

We can use the regression time series model

$$y_t = \beta_0 + \beta_1 x_t + R_t \ , \quad R_t = \phi R_{t-1} + \varepsilon_t$$

to forecast future observations, once we have estimated β_0, β_1, ϕ by least squares.
Want to forecast at $t=n+1,\ n+2, \ldots$

$$y_{t+1} = \beta_0 + \beta_1 x_{t+1} + R_{t+1} \ , \quad R_{t+1} = \phi R_t + \varepsilon_{t+1}$$

$$y_{t+1} = \beta_0 + \beta_1 x_{t+1} + \phi R_t + \varepsilon_{t+1}$$

which is

$$F_{n+1} = \hat{\beta}_0 + \hat{\beta}_1 x_{n+1} + \hat{\phi}\hat{R}_n \ , \qquad \hat{R}_n = y_n - (\hat{\beta}_0 + \hat{\beta}_1 x_n)$$

$$F_{t+2} = \hat{\beta}_0 + \hat{\beta}_1 x_{n+2} + \hat{\phi}\hat{R}_{n+1} \ , \quad \hat{R}_{n+1} = \phi \hat{R}_n \ , \quad \ldots$$

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Forecasting with Time Series Autoregressive Models

Approximate 95% PI Forecasting Limits

$$\hat{y}_{n+1} \pm 1.96\sqrt{MSE}$$

$$\hat{y}_{n+2} \pm 1.96\sqrt{MSE(1+\hat{\phi}^2)}$$

$$\hat{y}_{n+3} \pm 1.96\sqrt{MSE(1+\hat{\phi}^2+\hat{\phi}^4)}$$

.

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$$\hat{y}_{n+m} \pm 1.96\sqrt{MSE(1+\hat{\phi}^2+\hat{\phi}^4\cdots+\hat{\phi}^{2(m-1)})}$$

Better forecast than would be obtained using the standard least squares procedure.

Intro to Time Series Modeling & Forecasting

Fitting Time Series Models with Autoregressive Errors

SALES35.txt

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1	4.8	0.102857
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5	23.1	1.220336
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33	146.1	3.942689
34	151.4	4.947059
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Example: Data on annual sales y for 35 years. Use AR(1) model.

$$\hat{y}_t = .4058 + 4.2959t + 0.5896\hat{R}_{t-1}$$

a. Use the fitted model to forecast sales in years t=36, 37, and 38.

$$\hat{R}_{35} = y_{35} - \hat{\beta}_0 - \hat{\beta}_1(35) = 150.9 - 0.4058 - 4.2959(35) = 0.1377$$

$$F_{36} = \hat{\beta}_0 + \hat{\beta}_1(36) + \hat{\phi}\ \hat{R}_{35} = 0.4058 + 4.2959(36) + (0.5896)\ (0.1377) = 155.1394$$

$$F_{37} = \hat{\beta}_0 + \hat{\beta}_1(37) + \hat{\phi}^2\hat{R}_{35} = 0.4058 + 4.2959(37) + (0.5896)^2(0.1377) = 159.4020$$

$$F_{38} = \hat{\beta}_0 + \hat{\beta}_1(38) + \hat{\phi}^3\hat{R}_{38} = 0.4058 + 4.2959(38) + (0.5896)^3(0.1377) = 163.6782$$

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Fitting Time Series Models with Autoregressive Errors

Example: Data on annual sales y for 35 years. Use AR(1) model.

$\hat{y}_t = .4058 + 4.2959t + 0.5896\hat{R}_{t-1}$

b. Find an approximate 95% prediction interval for the forecasts.

$$MSE = 27.42767$$

$$F_{36} \pm 1.96\sqrt{MSE} \rightarrow 155.1394 \pm 1.96\sqrt{27.42767} \rightarrow 155.1394 \pm 10.2648$$

[144.8746,165.4042]

$$F_{37} \pm 1.96\sqrt{MSE(1+\phi^2)} \rightarrow 159.4020 \pm 1.96\sqrt{27.42767(1+0.5896^2)} \rightarrow 159.4020 \pm 11.9161$$

[147.4858,171.3181]

$$F_{38} \pm 1.96\sqrt{MSE(1+\phi^2+\phi^4)} \rightarrow 163.6782 \pm 1.96\sqrt{27.42767(1+0.5896^2+0.5896^4)} \rightarrow 163.6782 \pm 12.4389$$

[151.2393,176.1172]

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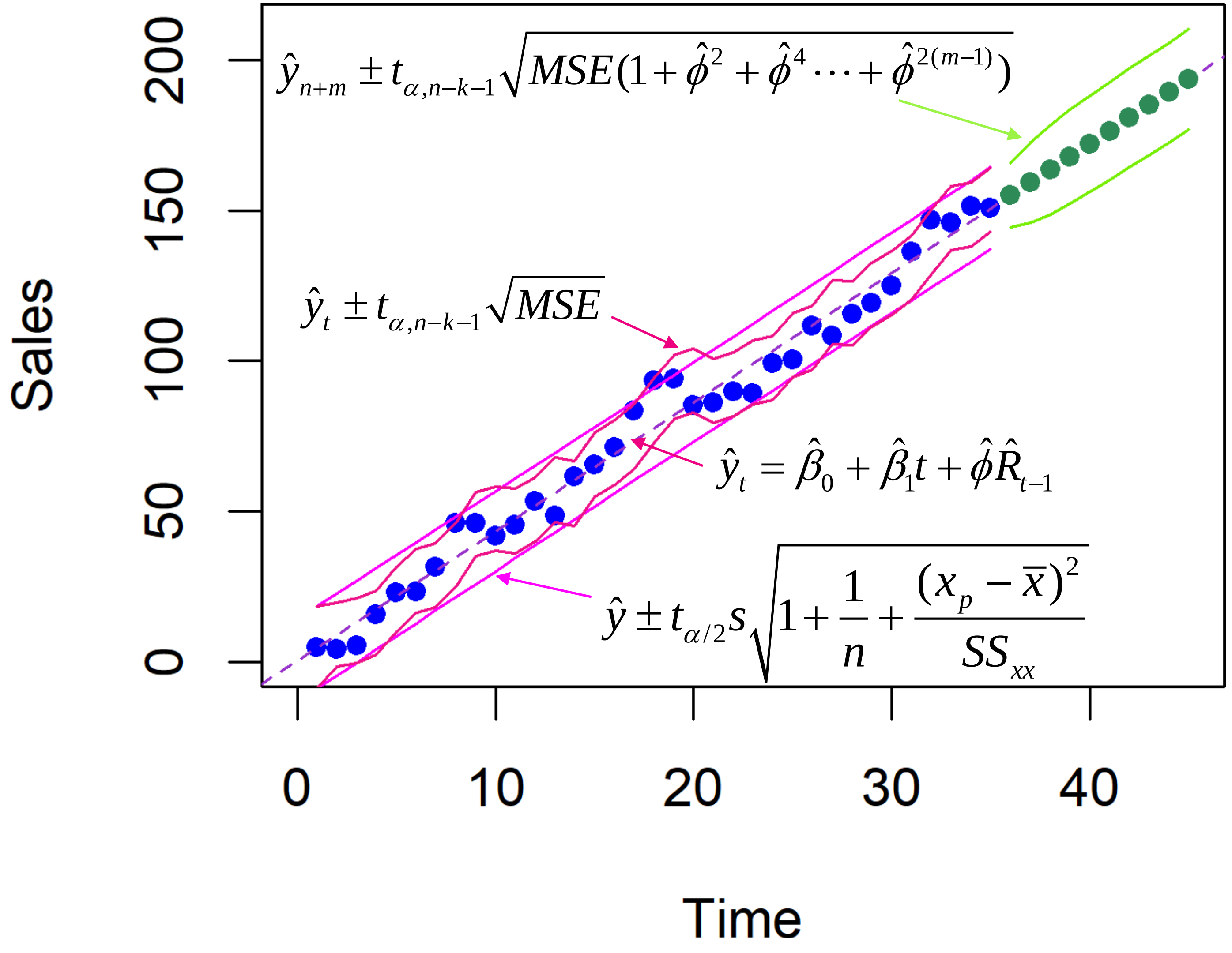
Forecasting with Time Series Autoregressive Models

Approximate 95% PI Forecasting Limits



Not Same
Best I could do

t	y	yhatUcor	FL	FU
1	4.8	4.697	-8.987	18.381
2	4	8.993	-4.634	22.619
3	5.5	13.288	-0.284	26.861
4	15.6	17.584	4.062	31.106
5	23.1	21.880	8.405	35.355
6	23.3	26.175	12.744	39.606
7	31.4	30.471	17.080	43.861
8	46	34.767	21.413	48.120
9	46.1	39.062	25.743	52.382
10	41.9	43.358	30.068	56.647
11	45.5	47.653	34.391	60.916
12	53.5	51.949	38.709	65.189
13	48.4	56.245	43.025	69.465
14	61.6	60.540	47.337	73.744
15	65.6	64.836	51.645	78.027
16	71.4	69.132	55.949	82.314
17	83.4	73.427	60.250	86.604
18	93.6	77.723	64.548	90.898
19	94.2	82.018	68.842	95.195
20	85.4	86.314	73.132	99.496
21	86.2	90.610	77.418	103.801
22	89.9	94.905	81.702	108.109
23	89.2	99.201	85.981	112.421
24	99.1	103.497	90.257	116.736
25	100	107.792	94.530	121.055
26	112	112.088	98.798	125.377
27	108	116.384	103.064	129.703
28	116	120.679	107.326	134.033
29	119	124.975	111.584	138.365
30	125	129.270	115.839	142.701
31	136	133.566	120.091	147.041
32	147	137.862	124.339	151.384
33	146	142.157	128.585	155.730
34	151	146.453	132.826	160.080
35	151	150.749	137.065	164.432
36	0	155.139	144.484	165.794
37	0	159.402	145.968	172.836
38	0	163.678	148.848	178.508
39	0	167.963	152.368	183.558
40	0	172.252	156.223	188.280
41	0	176.543	160.264	192.823
42	0	180.837	164.412	197.262
43	0	185.132	168.621	201.642
44	0	189.427	172.866	205.987
45	0	193.722	177.132	210.312

T	SALES	FORECAST	LCL95	UCL95
1	4.8	4.702	−10.644	20.048
2	4.0	9.056	−1.996	20.108
3	5.5	10.347	−0.673	21.367
4	15.6	12.994	2.004	23.984
5	23.1	20.712	9.750	31.675
6	23.3	26.897	15.961	37.834
7	31.4	28.778	17.865	39.692
8	46.0	35.317	24.425	46.210
9	46.1	45.689	34.815	56.563
10	41.9	47.511	36.653	58.368
11	45.5	46.797	35.954	57.641
12	53.5	50.683	39.851	61.515
13	48.4	57.163	46.340	67.985
14	61.6	55.919	45.103	66.734
15	65.6	65.465	54.654	76.275
16	71.4	69.586	58.778	80.394
17	83.4	74.769	63.961	85.577
18	93.6	83.607	72.797	94.417
19	94.2	91.384	80.570	102.199
20	85.4	93.501	82.680	104.322
21	86.2	90.075	79.245	100.906
22	89.9	92.310	81.468	103.152
23	89.2	96.254	85.399	107.110
24	99.1	97.605	86.733	108.477
25	100.3	105.205	94.315	116.095
26	111.7	107.675	96.765	118.586
27	108.2	116.160	105.226	127.094
28	115.5	115.859	104.900	126.818
29	119.2	121.927	110.940	132.913
30	125.2	125.871	114.855	136.887
31	136.3	131.172	120.124	142.220
32	146.8	139.480	128.398	150.561
33	146.1	147.434	136.316	158.551
34	151.4	148.784	137.628	159.940
35	150.9	153.672	142.475	164.868
36	.	155.140	143.901	166.379
37	.	159.403	145.743	173.062
38	.	163.679	148.875	178.484

Intro to Time Series Modeling & Forecasting

Fitting Time Series Models with Autoregressive Errors

```
# SALES35 #
nf   <-10
alph <-0.05
k    <-1
tcrit<-qt(1-alph/2,n-k-1)

# read data
mydata <- read.delim("SALES35.txt",header=TRUE,sep="",dec=".")

# parse out variables
n <- nrow(mydata)
t <- c(mydata[,1]) #T time
y <- c(mydata[,2]) #y sales
e <- c(mydata[,3]) #e ind model residual

## Uncorrelated model
# fit linear independent model
mymodel <- lm(y~t)
ee<-mymodel$residuals
cbind(e,ee) # e=ee
yhatUcor<-mymodel$fitted.values
yhatUcor
```

```
# Uncorrelated PI
yPIL <- matrix(rep(0,n),nrow=n,ncol=1)
yPIU <- matrix(rep(0,n),nrow=n,ncol=1)
sUcor<- summary(mymodel)$sigma

xbar<-mean(t)
SSxx<-sum(t^2)-(sum(t)^2/n)
for (i in 1:n){
  yPIL[i]<-yhatUcor[i]-tcrit*sUcor*sqrt(1+1/n+(t[i]-xbar)^2/SSxx)
  yPIU[i]<-yhatUcor[i]+tcrit*sUcor*sqrt(1+1/n+(t[i]-xbar)^2/SSxx)
  }
withinPI<-cbind(y,yhatUcor,yPIL,yPIU)
withinPI

## Correlated model
# values entered from book, slight roundoff
b0    <-0.4058
b1    <-4.2959
phihat<-0.5896
MSE   <-27.42767
```

Intro to Time Series Modeling & Forecasting

Fitting Time Series Models with Autoregressive Errors

```
Rt     <- matrix(rep(0,n),nrow=n,ncol=1)
for (i in 1:n){
  Rt[i]<-y[i]-b0-b1*i
}
yhatcor<- matrix(rep(0,n),nrow=n,ncol=1)
yhatcor[1]<-yhatUcor[1]# set first value to ucor
yhatcorL<- matrix(rep(0,n),nrow=n,ncol=1)
yhatcorU<- matrix(rep(0,n),nrow=n,ncol=1)
yhatcorL[1]<-yPIL[1]# set first value to ucor
yhatcorU[1]<-yPIU[1]# set first value to ucor
for (i in 2:n){
  yhatcor[i]<-b0+b1*t[i]+phihat*Rt[i-1]
  yhatcorL[i]<-yhatcor[i]-tcrit*sqrt(MSE)
  yhatcorU[i]<-yhatcor[i]+tcrit*sqrt(MSE)
}
cbind(y,Rt,yhatcor,yhatcorL,yhatcorU)

# forecast mean and interval
F  <- matrix(rep(0,nf),nrow=nf,ncol=1)
FL <- matrix(rep(0,nf),nrow=nf,ncol=1)
FU <- matrix(rep(0,nf),nrow=nf,ncol=1)
Rn <-y[n]-b0-b1*n
TMP    <-0
```

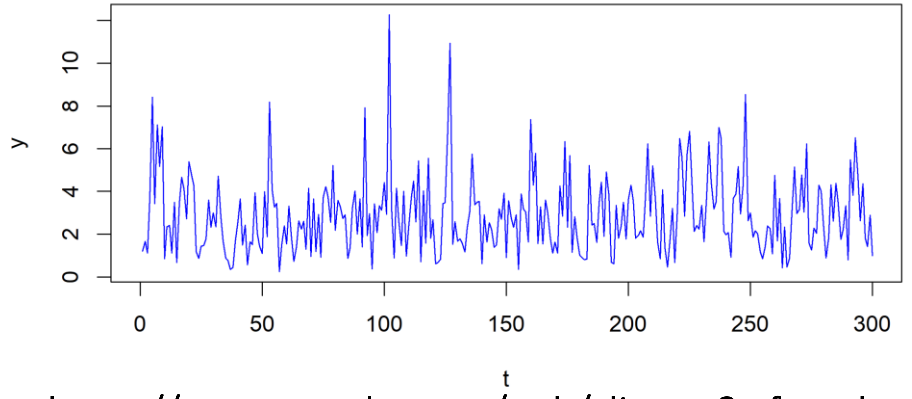
```
for (i in 1:nf){
  F[i] <-b0+b1*(n+i)+phihat^i*Rn
  TMP  <-TMP+phihat^(i-1)
  FL[i]<-F[i]-tcrit*sqrt(MSE*TMP)
  FU[i]<-F[i]+tcrit*sqrt(MSE*TMP)
}
Fall<-cbind(rep(0,nf),F,FL,FU)
Fall

# scatter plot with uncorrelated and correlated error lines
plot(t,y,pch=19,col="blue",xlab='Time',ylab='Sales',
    cex=0.75,xlim=c(0,n+nf),ylim=c(0,max(FU)))
abline(mymodel,lty=2,col='red')      # ind fit
lines(t,yPIL, type = "l", col = "magenta")
lines(t,yPIU, type = "l", col = "magenta")
abline(b0,b1,lty=2,col='darkorchid') # cor fit
points(seq(1,n,by=1),yhatcorL,col='deeppink2',type="l")
points(seq(1,n,by=1),yhatcorU,col='deeppink2',type="l")
points(seq(n+1,n+nf,by=1),F,pch=19,cex=0.75,col='seagreen')
points(seq(n+1,n+nf,by=1),FL,col='chartreuse2',type="l")
points(seq(n+1,n+nf,by=1),FU,col='chartreuse2',type="l")

results<-rbind(withinPI,Fall)
```

Intro to Time Series Modeling & Forecasting

Homework:
Read Chapter 10
Problems #: * (precipitationMKE), 27 (GOLDMON), 28

Submit at minimum one file with all your answers and another with your code.

* Fit a time series regression model to the MKE monthly precipitation data. Assume independent errors. Include all four components $y_t = T_t + C_t + S_t + R_t$. Examine residuals. Interpret.



https://www.weather.gov/wrh/climate?wfo=mkx

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2000	1.20	1.66	1.12	3.64	8.42	3.42	7.12	5.17	7.04	0.84	2.33	2.41
2001	1.11	3.48	0.67	3.45	4.68	4.13	2.70	5.41	4.76	4.29	1.19	0.86
2002	1.44	1.46	1.76	3.59	2.31	2.99	2.33	4.73	2.79	1.66	0.88	0.75
2003	0.35	0.43	1.64	2.61	3.65	1.49	2.43	0.57	1.65	1.51	3.94	2.03
2004	1.43	1.10	3.99	1.87	8.18	4.07	3.25	3.43	0.24	1.47	2.38	1.53
2005	3.31	1.79	0.72	1.41	2.62	2.23	2.60	1.29	4.17	0.95	3.65	1.18
2006	2.92	0.91	3.69	4.23	3.73	2.54	5.23	2.18	3.57	3.30	2.72	2.91
2007	0.86	1.36	3.21	4.02	1.99	3.64	1.40	7.92	1.93	2.96	0.36	3.41
2008	2.07	3.32	3.11	4.42	2.92	12.27	3.20	0.88	4.16	2.62	1.47	4.00
2009	0.97	2.29	3.68	4.50	2.56	5.44	0.71	4.04	1.57	5.57	1.80	2.68
2010	0.62	0.67	0.83	3.42	3.47	6.93	10.93	1.52	2.58	1.66	1.78	1.57
2011	1.16	2.30	3.08	5.75	3.37	3.48	3.53	0.62	2.91	1.63	2.53	2.23
2012	1.37	1.48	3.19	2.69	3.91	0.90	3.56	2.75	2.31	2.91	0.35	3.87
2013	3.17	3.03	1.63	7.38	4.30	5.80	1.55	3.27	1.54	3.59	2.97	1.79
2014	1.11	1.63	1.12	4.26	2.83	6.34	2.31	5.69	1.14	2.81	1.84	1.03
2015	0.91	0.81	0.83	5.22	2.43	2.49	1.60	3.46	4.44	1.90	4.93	3.82
2016	0.69	0.62	3.34	1.80	2.37	3.49	1.76	3.59	4.30	3.56	1.81	1.93
2017	2.16	1.85	3.72	6.23	2.83	5.21	3.69	1.63	0.85	4.09	1.37	0.46
2018	1.62	3.20	0.66	3.11	6.49	5.62	2.83	5.68	6.83	4.50	2.13	2.41
2019	2.23	3.35	1.64	3.77	6.32	4.42	3.17	3.53	7.00	6.48	2.14	1.96
2020	2.07	0.91	3.67	3.88	5.17	2.94	4.34	8.55	2.62	3.00	1.86	2.15
2021	2.02	1.19	0.84	1.41	2.38	2.25	1.07	4.77	1.68	3.67	0.42	2.34
2022	0.46	0.89	2.95	5.15	2.96	3.16	4.79	3.02	6.23	1.59	1.26	2.28
2023	2.03	4.30	4.02	2.24	0.88	1.82	4.33	2.60	4.40	3.42	1.75	2.22
2024	3.32	0.79	5.50	3.80	6.52	4.71	2.62	4.38	1.83	1.41	2.89	0.98

Intro to Time Series Modeling & Forecasting

Questions?