Chapter 6: Variable Screening Methods

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Variable Screening Methods

Introduction: Why Use a Variable Screening Method?

Researches often will collect a data set with a **large** number of independent variables, each of which is a potential predictor of some dependent variable, y.

The problem of deciding to include multiple regression model for $E(y)$ is common. Suppose y depends on 10 x's. 7 quantitative and 3 qualitative yield 288 terms.

$$\begin{aligned} E(y) = \beta_0 &+ \beta_1 x_1 + \ldots + \beta_7 x_7 + \beta_8 x_1 x_2 + \ldots + \beta_{28} x_6 x_7 \\ &+ \beta_{29} x_1^2 + \ldots + \beta_{35} x_7^2 + \beta_{36} x_8 + \ldots + \beta_{38} x_{10} \\ &+ \beta_{39} x_8 x_9 + \ldots + \beta_{42} x_8 x_9 x_{10} + \beta_{43} x_1 x_8 + \ldots + \beta_{77} x_7^2 x_8 \\ &+ \beta_{78} x_1 x_9 + \ldots + \beta_{112} x_7^2 x_9 + \ldots + \beta_{113} x_1 x_{10} + \ldots + \beta_{147} x_7^2 x_{10} \\ &+ \beta_{148} x_1 x_8 x_9 + \ldots + \beta_{182} x_7^2 x_8 x_9 + \beta_{183} x_1 x_8 x_{10} + \ldots + \beta_{217} x_7^2 x_8 x_{10} \\ &+ \beta_{218} x_1 x_9 x_{10} + \ldots + \beta_{252} x_7^2 x_9 x_{10} + \beta_{253} x_1 x_8 x_9 x_{10} + \ldots + \beta_{287} x_7^2 x_8 x_9 x_{10} \end{aligned}$$

Too complex to be practicably useful.

Variable Screening Methods

Stepwise Regression (Forward Selection)

Stepwise Regression: The user identifies the set of potentially important independent variables x's that influence the dependent (response) variable y.

Step 1: Fit all possible one-variable models of the form $E(y)=\beta_0+\beta_1 x_i$, $i=1,\ldots,k$.

Perform the t-test H_0: $\beta_1=0$ vs. H_a: $\beta_1\neq 0$.

$t=\hat{\beta}_i / s\sqrt{W_{ii}}$, W_{ii} is the i^{th} diagonal element of $W=(X'X)^{-1}$.

Select the best one variable model (largest $|t|$ statistic). Call it x_1

Step 2: Fit all two variable models with remaining x's, $E(y)=\beta_0+\beta_1 x_1+\beta_2 x_i$, $i\neq 1$.

Perform the t-test H_0: $\beta_2=0$ vs. H_a: $\beta_2\neq 0$.

$t=\hat{\beta}_i / s\sqrt{W_{ii}}$, W_{ii} is the i^{th} diagonal element of $W=(X'X)^{-1}$.

Select the best two variable model (largest $|t|$ statistic). Call it x_2

Go back and check the t-value of $\hat{\beta}_1$ after $\hat{\beta}_2$ has been added to the model.

Variable Screening Methods

Stepwise Regression (Forward Selection)

Step 3: Fit all three variable models with remaining x's, $E(y)=\beta_0+\beta_1 x_1+\beta_2 x_2 +\beta_3 x_i$, $i\neq 1,2$.

Perform the t-test H_0: $\beta_3=0$ vs. H_a: $\beta_3\neq 0$.

$t = \hat{\beta}_i / s\sqrt{W_{ii}}$, W_{ii} is the i^{th} diagonal element of $W=(X'X)^{-1}$.

Select the best two variable model (largest $|t|$ statistic). Call it x_2

Go back and check the t-values of $\hat{\beta}_1, \hat{\beta}_2$ after $\hat{\beta}_3$ has been added.

This procedure is continued until no further independent variables can be found that yield significant t-values (at the specified α level) in the presence of the variables already in the model.

Variable Screening Methods

Stepwise Regression (Forward Selection)

Example: Log salary y depends on 7 quantitative and 3 qualitative x variables. Which (linear) variables are important.

Independent Variable	Description
x_1	Experience (years)—quantitative
x_2	Education (years)—quantitative
x_3	Gender (1 if male, 0 if female)—qualitative
x_4	Number of employees supervised—quantitative
x_5	Corporate assets (millions of dollars)—quantitative
x_6	Board member (1 if yes, 0 if no)—qualitative
x_7	Age (years)—quantitative
x_8	Company profits (past 12 months, millions of dollars)—quantitative
x_9	Has international responsibility (1 if yes, 0 if no)—qualitative
x_{10}	Company's total sales (past 12 months, millions of dollars)—quantitative

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	11.091	.033		335.524	.000
	X1	.028	.002	.787	12.618	.000
2	(Constant)	10.968	.032		342.659	.000
	X1	.027	.002	.770	15.134	.000
	X3	.197	.028	.361	7.097	.000
3	(Constant)	10.783	.036		298.170	.000
	X1	.027	.001	.771	18.801	.000
	X3	.233	.023	.427	10.170	.000
	X4	.000	.000	.307	7.323	.000
4	(Constant)	10.278	.066		155.154	.000
	X1	.027	.001	.771	24.677	.000
	X3	.232	.017	.425	13.297	.000
	X4	.001	.000	.354	10.920	.000
	X2	.030	.004	.266	8.379	.000
5	(Constant)	9.962	.101		98.578	.000
	X1	.027	.001	.771	26.501	.000
	X3	.225	.016	.412	13.742	.000
	X4	.001	.000	.337	11.064	.000
	X2	.029	.003	.258	8.719	.000
	X5	.002	.000	.116	3.947	.000

R Code
Output →
t-statistics

Variable Screening Methods
Stepwise Regression (Forward Selection)

lm(formula = y ~ x1, data = df)
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 11.090887 0.033055 335.52 <2e-16 ***
x1 0.027839 0.002206 **12.62** <2e-16 ***
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.1612 on 98 degrees of freedom
Multiple R-squared: 0.619, Adjusted R-squared: 0.6151
F-statistic: 159.2 on 1 and 98 DF, p-value: < 2.2e-16

lm(formula = y ~ x2, data = df)
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 11.05594 0.17971 61.520 <2e-16 ***
x2 0.02491 0.01110 **2.243** 0.0271 *
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.2547 on 98 degrees of freedom
Multiple R-squared: 0.04884, Adjusted R-squared: 0.03914
F-statistic: 5.032 on 1 and 98 DF, p-value: 0.02713

lm(formula = y ~ x3, data = df)
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 11.31231 0.04112 275.116 < 2e-16 ***
x3 0.21623 0.05061 **4.272** 4.49e-05 ***
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.2398 on 98 degrees of freedom
Multiple R-squared: 0.157, Adjusted R-squared: 0.1484
F-statistic: 18.25 on 1 and 98 DF, p-value: 4.487e-05

lm(formula = y ~ x4, data = df)
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.134e+01 5.813e-02 195.157 <2e-16 ***
x4 3.236e-04 1.535e-04 **2.107** 0.0376 *
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.2554 on 98 degrees of freedom
Multiple R-squared: 0.04335, Adjusted R-squared: 0.03359
F-statistic: 4.441 on 1 and 98 DF, p-value: 0.03763

Call:
lm(formula = y ~ x5, data = df)
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 10.853365 0.293139 37.02 <2e-16 ***
x5 0.003436 0.001668 **2.06** 0.042 *
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.2557 on 98 degrees of freedom
Multiple R-squared: 0.04152, Adjusted R-squared: 0.03174
F-statistic: 4.245 on 1 and 98 DF, p-value: 0.04202

lm(formula = y ~ x6, data = df)
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 11.46777 0.03652 314.017 <2e-16 ***
x6 -0.02603 0.05217 **-0.499** 0.619
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.2608 on 98 degrees of freedom
Multiple R-squared: 0.002533, Adjusted R-squared: -0.007645
F-statistic: 0.2489 on 1 and 98 DF, p-value: 0.619

lm(formula = y ~ x7, data = df)
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 10.682669 0.098393 108.571 < 2e-16 ***
x7 0.018029 0.002247 **8.022** 2.28e-12 ***
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.2029 on 98 degrees of freedom
Multiple R-squared: 0.3964, Adjusted R-squared: 0.3902
F-statistic: 64.35 on 1 and 98 DF, p-value: 2.277e-12

lm(formula = y ~ x8, data = df)
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 11.388078 0.132479 85.961 <2e-16 ***
x8 0.008693 0.016868 **0.515** 0.607
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.2608 on 98 degrees of freedom
Multiple R-squared: 0.002703, Adjusted R-squared: -0.007474
F-statistic: 0.2656 on 1 and 98 DF, p-value: 0.6075

lm(formula = y ~ x9, data = df)
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 11.45432 0.02884 397.211 <2e-16 ***
x9 0.00386 0.06797 **0.057** 0.955
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.2611 on 98 degrees of freedom
Multiple R-squared: 3.291e-05, Adjusted R-squared: -0.01017
F-statistic: 0.003225 on 1 and 98 DF, p-value: 0.9548

lm(formula = y ~ x10, data = df)
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 11.325878 0.238765 47.435 <2e-16 ***
x10 0.005201 0.009558 **0.544** 0.588
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.2607 on 98 degrees of freedom
Multiple R-squared: 0.003012, Adjusted R-squared: -0.007161
F-statistic: 0.2961 on 1 and 98 DF, p-value: 0.5876

Variable Screening Methods

Stepwise Regression Most look at R^2 or R_a^2 instead of t-statistic.

	Index	N	Predictors	R-Square	Adj. R-Square	Mallow's Cp
1	1	1	x1	0.6189794572	0.615091492	343.85658
7	2	1	x7	0.3963753117	0.390215876	600.83459
3	3	1	x3	0.1570025883	0.148400574	877.17052
2	4	1	x2	0.0488408794	0.039135174	1002.03423
4	5	1	x4	0.0433532791	0.033591578	1008.36921
5	6	1	x5	0.0415163660	0.031735921	1010.48978
10	7	1	x10	0.0030120259	-0.007161321	1054.93984
8	8	1	x8	0.0027029290	-0.007473572	1055.29667
6	9	1	x6	0.0025329167	-0.007645319	1055.49293
9	10	1	x9	0.0000329115	-0.010170834	1058.37898
12	11	2	x1 x3	0.7492074614	0.744036481	195.51916
11	12	2	x1 x2	0.6676461635	0.660793507	289.67491
13	13	2	x1 x4	0.6657473331	0.658855526	291.86695
14	14	2	x1 x5	0.6591000500	0.652071185	299.54069
15	15	2	x1 x6	0.6258493043	0.618134857	337.92591
19	16	2	x1 x10	0.6251626373	0.617434032	338.71861
17	17	2	x1 x8	0.6211730639	0.613362199	343.32425
16	18	2	x1 x7	0.6202405195	0.612410427	344.40079
18	19	2	x1 x9	0.6195497268	0.611705391	345.19825
31	20	2	x3 x7	0.5097037383	0.499594537	472.00633
24	21	2	x2 x7	0.4647957052	0.453760565	523.84892
42	22	2	x5 x7	0.4466738810	0.435265095	544.76906
37	23	2	x4 x7	0.4294675355	0.417703979	564.63236
52	24	2	x7 x10	0.3992410018	0.386854218	599.52639
50	25	2	x7 x8	0.3976158033	0.385195511	601.40254

	Index	N	Predictors	R-Square	Adj. R-Square	Mallow's Cp
46	26	2	x6 x7	0.3974306886	0.385006579	601.61624
51	27	2	x7 x9	0.3973609447	0.384935397	601.69676
28	28	2	x3 x4	0.2466304387	0.231097046	775.70261
20	29	2	x2 x3	0.1986675946	0.182145277	831.07173
29	30	2	x3 x5	0.1858378993	0.169051052	845.88255
34	31	2	x3 x10	0.1722383501	0.155171100	861.58210
30	32	2	x3 x6	0.1661114504	0.148917872	868.65510
32	33	2	x3 x8	0.1583617684	0.141008403	877.60146
33	34	2	x3 x9	0.1576238634	0.140255283	878.45331
21	35	2	x2 x4	0.1123065965	0.094003640	930.76833
22	36	2	x2 x5	0.0864422276	0.067605985	960.62660
35	37	2	x4 x5	0.0766236732	0.057584986	971.96131
27	38	2	x2 x10	0.0510848803	0.031519620	1001.44372
25	39	2	x2 x8	0.0503306478	0.030749836	1002.31442
23	40	2	x2 x6	0.0502055536	0.030622163	1002.45883
26	41	2	x2 x9	0.0489118938	0.029301830	1003.95225
40	42	2	x4 x10	0.0472231622	0.027578279	1005.90175
38	43	2	x4 x8	0.0460452751	0.026376105	1007.26153
45	44	2	x5 x10	0.0437062688	0.023988872	1009.96172
39	45	2	x4 x9	0.0433907498	0.023666848	1010.32596
36	46	2	x4 x6	0.0433809373	0.023656833	1010.33728
44	47	2	x5 x9	0.0421443980	0.022394798	1011.76477
41	48	2	x5 x6	0.0419712177	0.022218047	1011.96469
43	49	2	x5 x8	0.0417417885	0.021983887	1012.22955
54	50	2	x8 x10	0.0057435143	-0.014756619	1053.78656
49	51	2	x6 x10	0.0052147962	-0.015296239	1054.39693

Variable Screening Methods

Stepwise Regression

```
# R Code
# read data
mydata<-read.delim("execsal.txt",header=FALSE,sep="",dec=".")
# parse out variables
n <- nrow(mydata)
k <- ncol(mydata)-1

# Parse all variables
y  <- c(mydata[, 1]) #ln salary
x1 <- c(mydata[, 2]) #x1
x2 <- c(mydata[, 3]) #x2
x3 <- c(mydata[, 4]) #x3
x4 <- c(mydata[, 5]) #x4
x5 <- c(mydata[, 6]) #x5
x6 <- c(mydata[, 7]) #x6
x7 <- c(mydata[, 8]) #x7
x8 <- c(mydata[, 9]) #x8
x9 <- c(mydata[,10]) #x9
x10<- c(mydata[,11]) #x10
df<- data.frame(cbind(x1,x2,x3,x4,x5,x6,x7,x8,x9,x10))
names(df)<-c("x1","x2","x3","x4","x5","x6","x7","x8","x9","x10")
```

```
# one at a time fits
lmx1  <- lm(y~x1,data=df)
summary.lm(lmx1)
lmx2  <- lm(y~x2,data=df)
summary.lm(lmx2)
lmx3  <- lm(y~x3,data=df)
summary.lm(lmx3)
lmx4  <- lm(y~x4,data=df)
summary.lm(lmx4)
lmx5  <- lm(y~x5,data=df)
summary.lm(lmx5)
lmx6  <- lm(y~x6,data=df)
summary.lm(lmx6)
lmx7  <- lm(y~x7,data=df)
summary.lm(lmx7)
lmx8  <- lm(y~x8,data=df)
summary.lm(lmx8)
lmx9  <- lm(y~x9,data=df)
summary.lm(lmx9)
lmx10 <- lm(y~x10,data=df)
summary.lm(lmx10)
```

```
# use stepwise function
install.packages("olsrr")
library(olsrr)
model = lm(y~.,data=df)
k=ols_step_all_possible(model,max_order=3)
k
```

Variable Screening Methods

All-Possible-Regressions Selection Procedure

There are several criteria that can be used.

1. $R^2 = 1 - \frac{SSE}{SS(Total)}$

R-Square

2. $R_a^2 = 1 - (n-1)\left[\frac{MSE}{SS(Total)}\right]$

Adjusted R-Square

3. $C_p = \frac{SSE_p}{MSE_k} + 2(p+1) - n$

Mallow's C_p

4. $PRESS = \sum_{i=1}^{n} [y_i - \hat{y}_{(i)}]^2$

Predictive Sum of Squares

1. R^2 criterion
   Looking for a simple model that is as good as, or nearly as good as, the model with all k independent variables.
2. Adjusted R^2 or MSE criterion
   Prefer the model with largest, or near largest, adjusted R^2.
3. Mallow's C_p Criterion
   Prefer a small value of C_p and a value of C_p near p+1.
4. *PRESS* Criterion
   Desire a model with a small *PRESS*.

Variable Screening Methods

All-Possible-Regressions Selection Procedure

There are several criteria that can be used.

1. $R^2 = 1 - \dfrac{SSE}{SS(Total)}$

R-Square

2. $R_a^2 = 1 - (n-1)\left[\dfrac{MSE}{SS(Total)}\right]$

Adjusted R-Square

3. $C_p = \dfrac{SSE_p}{MSE_k} + 2(p+1) - n$

Mallow's C_p

4. $PRESS = \sum_{i=1}^{n} [y_i - \hat{y}_{(i)}]^2$

Predictive Sum of Squares

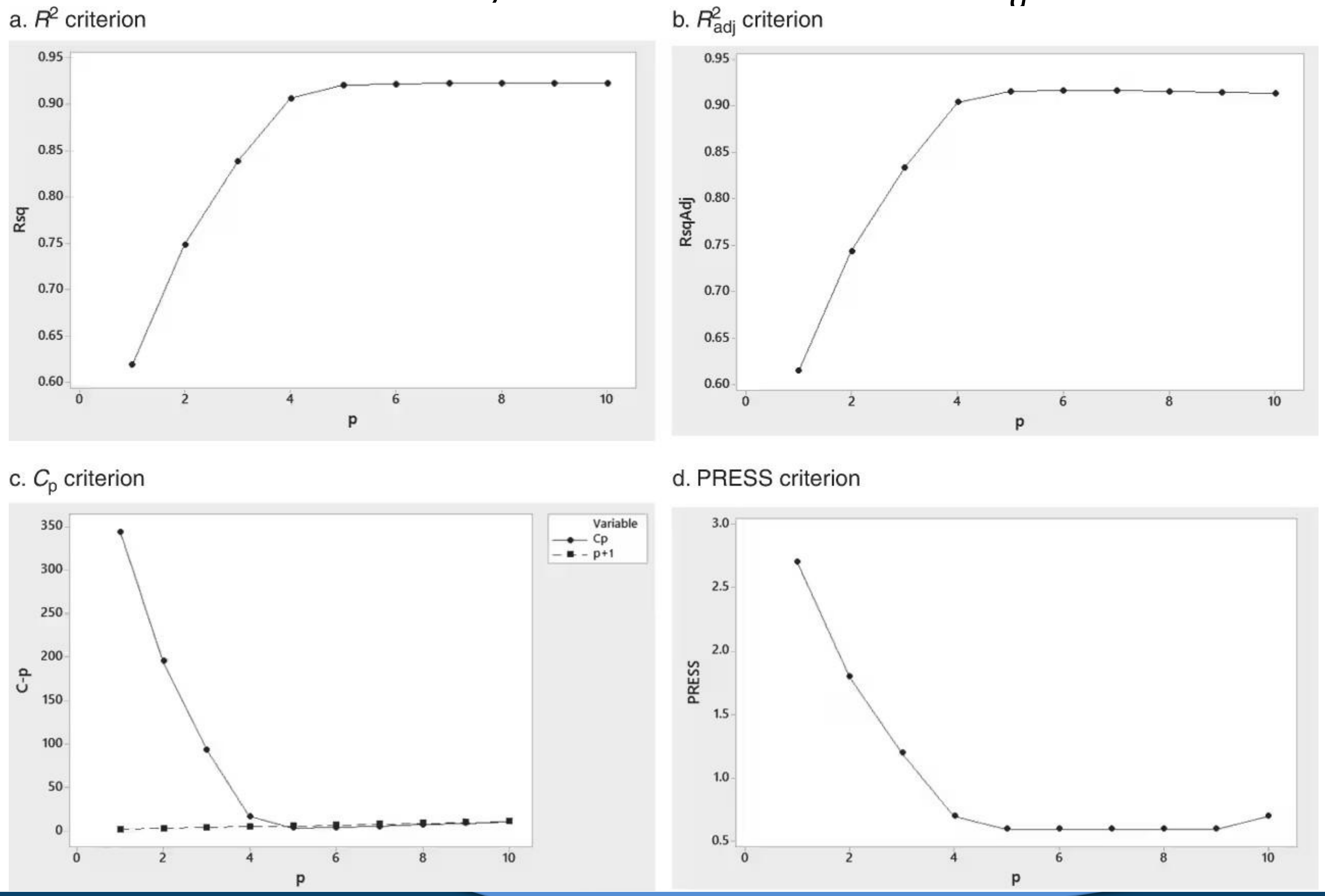
Number of Predictors p	Variables in the Model	R^2	adj-R^2	MSE	C_p	PRESS
1	x_1	.619	.615	.0260	343.9	2.664
2	x_1, x_3	.749	.744	.0173	195.5	1.788
3	x_1, x_3, x_4	.839	.834	.0112	93.8	1.171
4	x_1, x_2, x_3, x_4	.907	.904	.0065	16.8	.696
5	x_1, x_2, x_3, x_4, x_5	.921	.916	.0056	3.6	.610
6	$x_1, x_2, x_3, x_4, x_5, x_9$	.922	.917	.0056	4.0	.610
7	$x_1, x_2, x_3, x_4, x_5, x_6, x_9$	.923	.917	.0056	5.4	.620
8	$x_1, x_2, x_3, x_4, x_5, x_6, x_8, x_9$	.923	.916	.0057	7.2	.629
9	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9$	.923	.915	.0057	9.1	.643
10	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}$	.923	.914	.0058	11.0	.654

The best one-variable model includes x_1 (years of experience), the best two-variable model includes x_1 and x_3 (gender), the best three-variable model includes x_1, x_3, x_4 (number supervised) and so on.

Variable Screening Methods

All-Possible-Regressions Selection Procedure

Instead of *t* statistic, most look at R^2 or R_a^2.



According to all four criteria, the variables x_1, x_2, x_3, x_4, and x_5 should all be included.

Variable Screening Methods

Homework:

Read Chapter 6

Problems #: A data frame (mtcars) with n=32 observations on 11 variables.
Use mpg as y and cyl, disp, hp, drat, wt, qsec, vs, am, gear, carb as x_1-x_9. Hypothesize the form of a model, $E(y)$.
Perform model building to select variables to determine a good model.

y	mpg	Miles/(US) gallon
x_1	cyl	Number of cylinders (4,6,8)
x_2	disp	Displacement (cu.in.)
x_3	hp	Gross horsepower
x_4	drat	Rear axle ratio
x_5	wt	Weight (1000 lbs)
x_6	qsec	1/4 mile time (seconds)
x_7	vs	Engine (0 = V-shaped, 1 = straight)
x_8	am	Transmission (0 = automatic, 1 = manual)
x_9	gear	Number of forward gears (1,2,...,8)

Variable Screening Methods

Questions?