

Class 13

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Agenda:

Recap Chapter 8.1 - 8.2

Lecture Chapter 8.3 - 8.5

Recap Chapter 8.1-8.2

8: Introduction to Statistical Inference

8.1 The Nature of Estimation

Point estimate for a parameter: A single number ..., to estimate a parameter ... usually the .. **sample statistic.**

i.e. $\bar{X}$ is a point estimate for μ

Interval estimate: An interval bounded by two values and used to estimate the value of a population parameter.

i.e. $\bar{x} \pm (\text{some amount})$ is an interval estimate for μ .

point estimate $\pm$ some amount

8: Introduction to Statistical Inference

8.1 The Nature of Estimation

Significance Level: Probability parameter outside interval, α .

$$P(\mu \text{ not in } \bar{x} \pm \text{some amount}) = \alpha$$

Level of Confidence $1-\alpha$:

$$P(\bar{x} - \text{some amount} < \mu < \bar{x} + \text{some amount}) = 1 - \alpha$$

Confidence Interval:

point estimator $\pm$ some amount that depends on
confidence level

8: Introduction to Statistical Inference

8.2 Estimation of Mean μ (σ Known)

By SDSM

$$\mu_{\bar{x}} = \mu, \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

What this implies is that $z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$

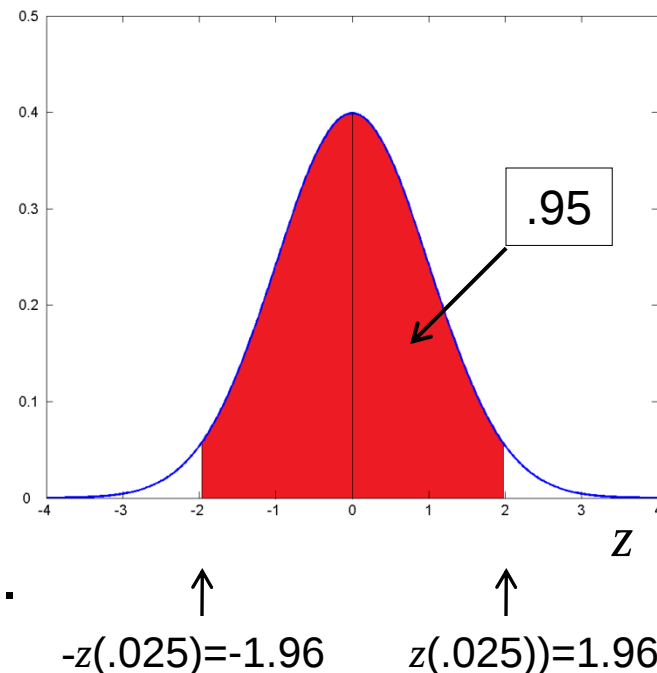
has an approximate standard normal distribution!

$$P(-1.96 < z < 1.96) = 0.95 \quad \alpha = .05$$

Or more generally,

$$P(-z(\alpha / 2) < z < z(\alpha / 2)) = 1 - \alpha$$

$z(\alpha / 2)$ called the confidence coefficient.



8: Introduction to Statistical Inference

8.2 Estimation of Mean μ (σ Known)

Thus, a $(1-\alpha)\times 100\%$ confidence interval for μ is

$$\bar{x} \pm z(\alpha / 2) \frac{\sigma}{\sqrt{n}}$$

which if $\alpha=0.05$, a 95% confidence interval for μ is

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}} \quad z(.025)=1.96$$

Confidence Interval for Mean:

$$\bar{x} - z(\alpha / 2) \frac{\sigma}{\sqrt{n}} \quad \text{to} \quad \bar{x} + z(\alpha / 2) \frac{\sigma}{\sqrt{n}} \quad (8.1)$$

Chapter 8: Introduction to Statistical Inference

Questions?

Homework: Read Chapter 8.1-8.2

WebAssign

Chapter 8 # 5, 15, 22, 24, 35, 47

Lecture Chapter 8.3 - 8.5

8: Introduction to Statistical Inference

8.3 The Nature of Hypothesis Testing

We make decisions every day in our lives.

Should I do A or should I do B (not A)?

Hypothesis: A statement that something is true.

Example:

Proposed Hypothesis:

“The party will be a great time.”

Opposing Hypothesis:

“The party will be a dud.”

8: Introduction to Statistical Inference

8.3 The Nature of Hypothesis Testing

How do you decide whether to go?

In order to determine which hypothesis we believe is true,
we perform something called a **statistical hypothesis test**.

Statistical hypothesis test: A process by which a decision is made between two opposing hypotheses. ... (read the rest)

8: Introduction to Statistical Inference

8.3 The Nature of Hypothesis Testing

We call the proposed hypothesis the **null hypothesis** and the opposing hypothesis the **alternative hypothesis**.

Null Hypothesis, H_0 : The hypothesis that we will test. Generally a statement that a parameter has a specific value.

Alternative Hypothesis, H_a : A statement about the same parameter that is used in the null hypothesis. ... parameter has a value different ... from the value in the null hypothesis.

8: Introduction to Statistical Inference

8.3 The Nature of Hypothesis Testing

Example 1: Friend's Party.

H_0 : "The party will be a dud"

VS.

H_a : "The party will be a great time"

8: Introduction to Statistical Inference

8.3 The Nature of Hypothesis Testing

Example 1: Friend's Party.

H_0 : "The party will be a dud"

VS.

H_a : "The party will be a great time"

Example 2: Math 1700 Students Height

H_0 : The mean height of Math 1700 students is 69", $\mu = 69$ ".

VS.

H_a : The mean height of Math 1700 students is not 69", $\mu \neq 69$ ".

8: Introduction to Statistical Inference

8.3 The Nature of Hypothesis Testing

Example 1: Friend's Party

H_0 : "The party will be a dud"

vs.

H_a : "The party will be a great time"

Four outcomes from a hypothesis test.

	Party Great	Party a dud.
We go.	Correct Decision	Error
We do not go.	Error	Correct Decision

There is always a chance we make an error.

8: Introduction to Statistical Inference

8.3 The Nature of Hypothesis Testing

Example 1: Friend's Party

H_0 : "The party will be a dud"

vs.

H_a : "The party will be a great time"

If do not go to party and it's great,
we made an error in judgment.

If go to party and it's a dud,
we made in error in judgment.

Four outcomes from a hypothesis test.

	Party Great	Party a dud.
We go.	Correct Decision	Error
We do not go.	Error	Correct Decision

There is always a chance
we make an error.

8: Introduction to Statistical Inference

8.3 The Nature of Hypothesis Testing

Example 2: Math 1700 Height

$$H_0: \mu = 69''$$

vs.

$$H_a: \mu \neq 69''$$

Four outcomes from a hypothesis test.

	$\mu = 69$	$\mu \neq 69$
Fail to reject H_0 .	Correct Decision	Error
Reject H_0 .	Error	Correct Decision

There is always a chance we make an error.

8: Introduction to Statistical Inference

8.3 The Nature of Hypothesis Testing

Example 2: Math 1700 Height

$H_0: \mu = 69$ "

vs.

$H_a: \mu \neq 69$ "

If we reject H_0 and it is true,
we made in error in judgment.

If we do not reject H_0 and it is false,
we have made an error in judgment.

Four outcomes from a hypothesis test.

	$\mu = 69$	$\mu \neq 69$
Fail to reject H_0 .	Correct Decision	Error
Reject H_0 .	Error	Correct Decision

There is always a chance
we make an error.

8: Introduction to Statistical Inference

8.3 The Nature of Hypothesis Testing

Type I Error: When a true null hypothesis H_0 is rejected.

Type II Error: When we decide in favor of a null hypothesis that is actually false.

Four outcomes from
a hypothesis test.

	H_0 True	H_0 False
Fail to Reject H_0	Type A Correct Decision	Type II Error
Reject H_0	Type I Error	Type B Correct Decision

8: Introduction to Statistical Inference

8.3 The Nature of Hypothesis Testing

Level of Significance (α): The probability of committing a type I error. (Sometimes α is called the false positive rate.)

	H_0 True	H_0 False
Fail to Reject H_0	Type A Correct Decision ($1-\alpha$)	Type II Error (β)
Reject H_0	Type I Error (α)	Type B Correct Decision ($1-\beta$)

8: Introduction to Statistical Inference

8.3 The Nature of Hypothesis Testing

Level of Significance (α): The probability of committing a type I error. (Sometimes α is called the false positive rate.)

Type II Probability (β):
The probability of committing a type II error.
(Sometimes β is called the false negative rate.)

	H_0 True	H_0 False
Fail to Reject H_0	Type A Correct Decision ($1-\alpha$)	Type II Error (β)
Reject H_0	Type I Error (α)	Type B Correct Decision ($1-\beta$)

8: Introduction to Statistical Inference

8.3 The Nature of Hypothesis Testing

We need to determine a measure that will quantify what we should believe.

Test Statistic: A random variable whose value is calculated from the sample data and is used in making the decision “reject H_0 : or “fail to reject H_0 .”

8: Introduction to Statistical Inference

8.3 The Nature of Hypothesis Testing

We need to determine a measure that will quantify what we should believe.

Test Statistic: A random variable whose value is calculated from the sample data and is used in making the decision “reject H_0 : or “fail to reject H_0 .”

Example 1: Friend's Party

Fraction of friends parties that were good.

Example 2: Math 1700 Heights

Sample mean height.

8: Introduction to Statistical Inference

8.4 Hypothesis Test of Mean (σ Known): A Probability-Value Approach

Note the first item presented in this section!

The assumption for hypothesis tests about mean μ using known σ : The sampling distribution of $\bar{x}$ has a normal distribution.

Use normal distribution for $\bar{x}$ to know when we have an unusual one compared to what we think it is.

8: Introduction to Statistical Inference

8.4 Hypothesis Test of Mean (σ Known): Probability Approach

THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

Step 1 The Set-Up:

Step 2 The Hypothesis Test Criteria:

Step 3 The Sample Evidence:

Step 4 The Probability Distribution:

Step 5 The Results:

8: Introduction to Statistical Inference

8.4 Hypothesis Test of Mean (σ Known): Probability Approach

THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

Step 1 The Set-Up:

- a. Describe the population parameter of interest.

The population parameter of interest is the mean μ , the height of Math 1700 students.

8: Introduction to Statistical Inference

8.4 Hypothesis Test of Mean (σ Known): Probability Approach

THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

Step 1 The Set-Up:

- b. State the null hypothesis (H_0) and the alternative hypotheses (H_a).

Null Hypothesis	Alternative Hypothesis
1. Greater than or equal to ($\geq$)	Less than ($<$)
2. Less than or equal to ($\leq$)	Greater than ($>$)
3. Equal to ($=$)	Not equal to ($\neq$)

$H_0: \mu = 69''$ vs. $H_a: \mu \neq 69''$

Figure from Johnson & Kubby, 2012.

8: Introduction to Statistical Inference

8.4 Hypothesis Test of Mean (σ Known): Probability Approach

THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

Step 2 The Hypothesis Test Criteria:

- a. Check the assumptions.

Assume we know σ from past experience.

Assume that n is “large” so that by the CLT $\bar{x}$ is normally distributed.

$$\mu_{\bar{x}} = \mu_0, \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Hypothesized value.

8: Introduction to Statistical Inference

8.4 Hypothesis Test of Mean (σ Known): Probability Approach

THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

Step 2 The Hypothesis Test Criteria:

- b. Identify the probability distribution and the test statistic to be used.

The standard normal distribution is to be used because $\bar{x}$ is expected to have a normal distribution.

Test Statistic for Mean:

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad \text{where } \sigma \text{ is assumed to be known} \quad (8.4)$$

Hypothesized value.

8: Introduction to Statistical Inference

8.4 Hypothesis Test of Mean (σ Known): Probability Approach

THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

Step 3 The Sample Evidence:

- a. Collect a sample of information.

Take a random sample from a population with mean μ that being questioned.

- b. Calculate the value of the test statistic.

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{67.2 - 69}{4 / \sqrt{15}} = -1.74$$

Assuming $n=15$ and 67.2 is sample mean. With known $\sigma = 4$.

8: Introduction to Statistical Inference

8.4 Hypothesis Test of Mean (σ Known): Probability Approach

THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

Step 4 The Probability Distribution:

- a. Calculate the p -value for the test statistic.

Probability value, or p -value: The probability that the test statistic could be the value it is or a more extreme value.

$$H_0: \mu = 69'' \text{ vs. } H_a: \mu \neq 69''$$

$$P(z > |z^*|) = p\text{-value}$$

since two sided test.

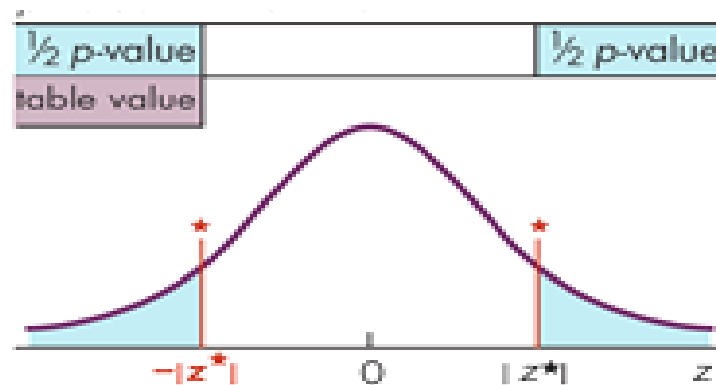


Figure from Johnson & Kuby, 2012.

8: Introduction to Statistical Inference

8.4 Hypothesis Test of Mean (σ Known): Probability Approach

THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

Step 5 The Results:

- a. State the decision about H_0 .

Is the p -value small enough to show that the sample evidence is highly unlikely if the null hypothesis were true?

Decision rule:

- a. If the p -value is *less than or equal to* the level of significance α , then the decision must be **reject H_0** .
- b. If the p -value is *greater than* the level of significance α , then the decision must be **fail to reject H_0** .

8: Introduction to Statistical Inference

8.4 Hypothesis Test of Mean (σ Known): Probability Approach

THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

Step 5 The Results:

b. State the conclusion about H_a .

With $\alpha = 0.05$, $p\text{-value} = 0.0819$.

Therefore there is not sufficient evidence to reject H_0 .

Fail to reject H_0 .

$$P(z < -1.74) = 0.0409$$

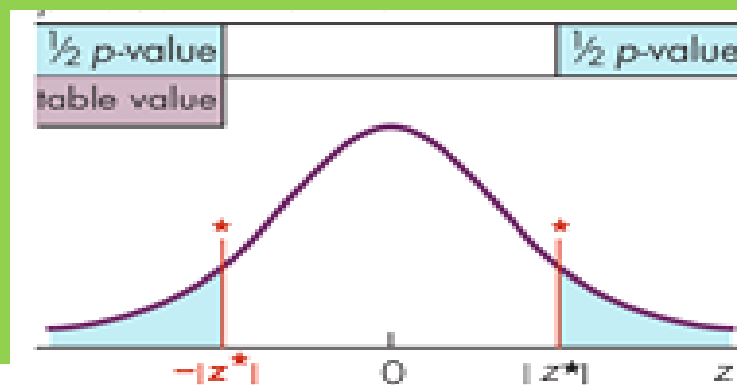


Figure from Johnson & Kuby, 2008.

8: Introduction to Statistical Inference

8.5 Hypothesis Test of Mean μ (σ Known):

A Classical Approach

THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

Step 1 The Set-Up:

Step 2 The Hypothesis Test Criteria:

Step 3 The Sample Evidence:

Step 4 The Probability Distribution:

Step 5 The Results:

8: Introduction to Statistical Inference

8.5 Hypothesis Test of Mean (σ Known): Classical Approach

THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

Step 1 The Set-Up:

- a. Describe the population parameter of interest.

The population parameter of interest is the mean μ , the height of Math 1700 students.

8: Introduction to Statistical Inference

8.5 Hypothesis Test of Mean (σ Known): Probability Approach

THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

Step 1 The Set-Up:

- b. State the null hypothesis (H_0) and the alternative hypotheses (H_a).

Null Hypothesis	Alternative Hypothesis
1. Greater than or equal to ($\geq$)	Less than ($<$)
2. Less than or equal to ($\leq$)	Greater than ($>$)
3. Equal to ($=$)	Not equal to ($\neq$)

$H_0: \mu = 69''$ vs. $H_a: \mu \neq 69''$

Figure from Johnson & Kubby, 2012.

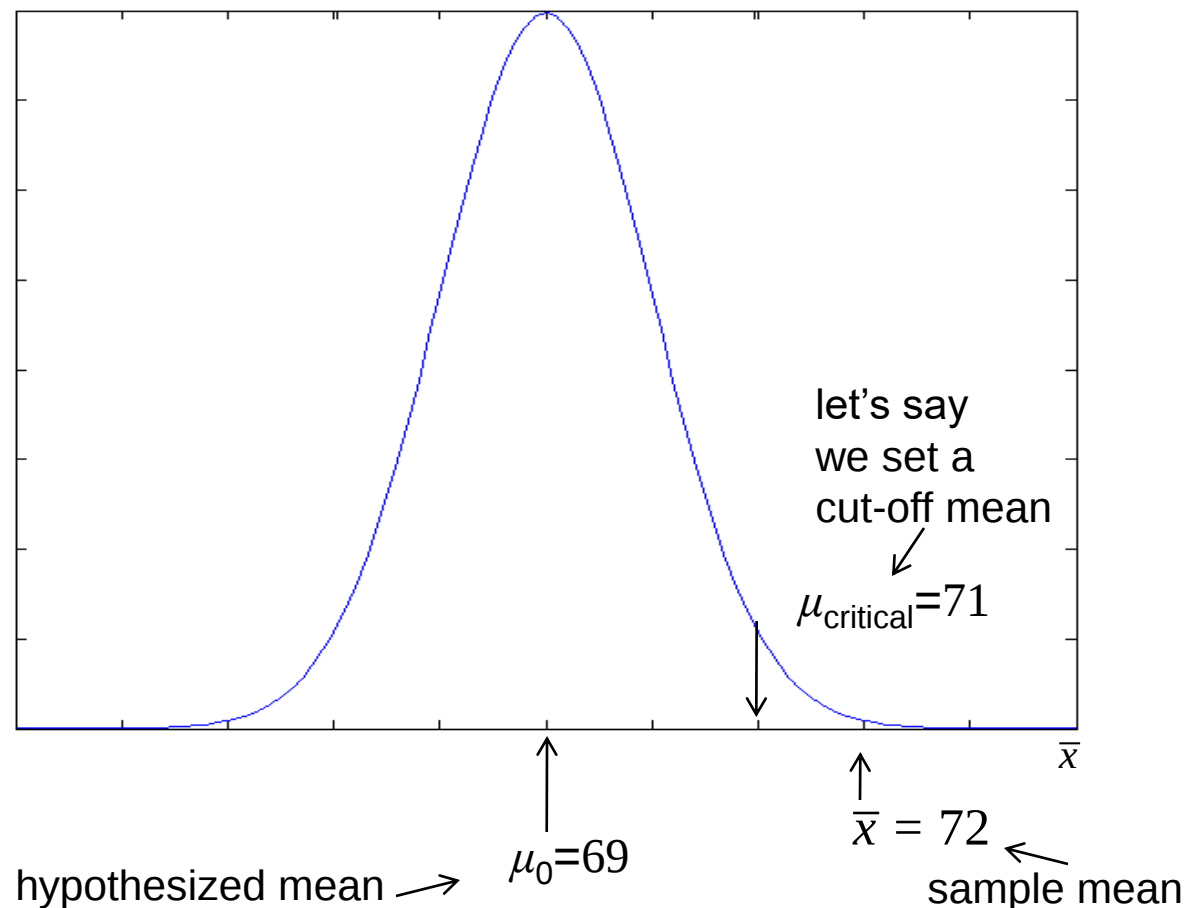
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8.5 Hypothesis Test of Mean (σ Known): Classical Approach

Scenario:

True μ not likely
 $\mu_0=69$ ".

Reject $H_0: \mu=\mu_0$
(do not believe H_0)



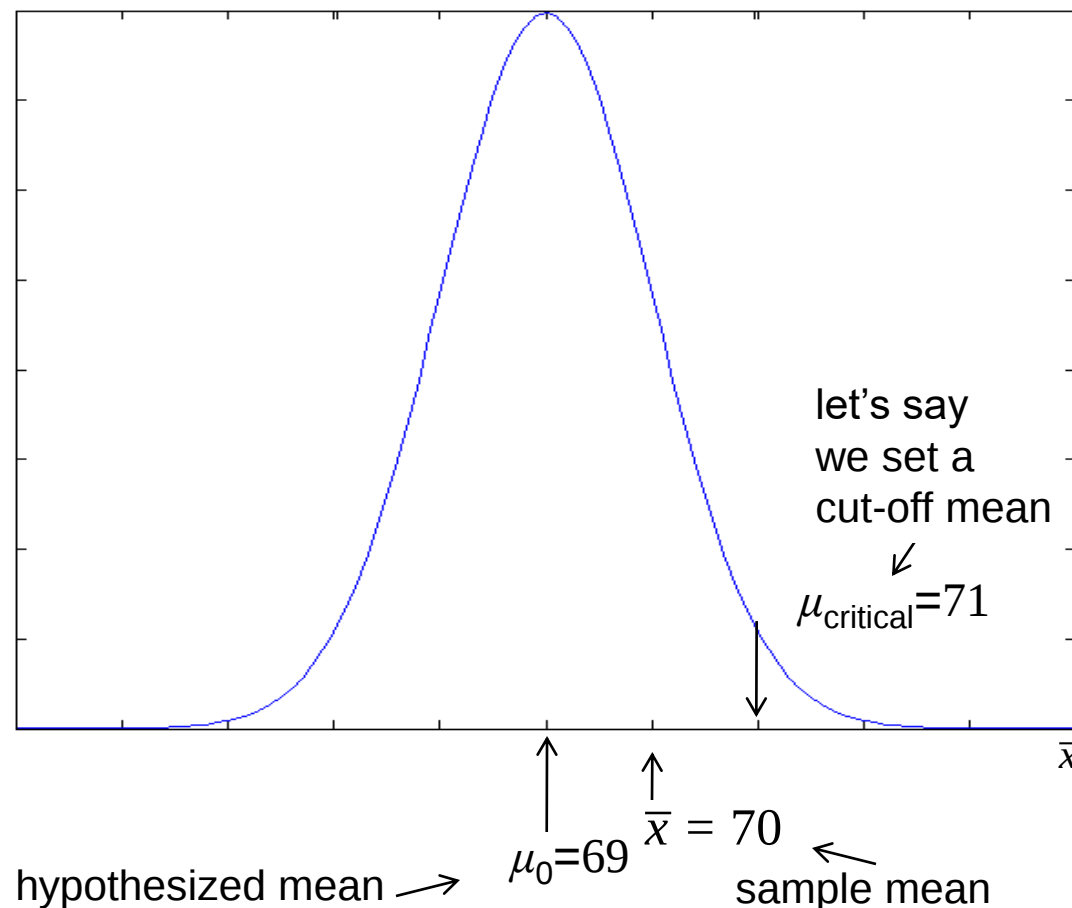
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8.5 Hypothesis Test of Mean (σ Known): Classical Approach

Scenario:

True μ likely
 $\mu_0=69$ ".

Fail to reject $H_0: \mu=\mu_0$
(not enough evidence
not to believe H_0)



8: Introduction to Statistical Inference

8.5 Hypothesis Test of Mean (σ Known): Classical Approach

We need a “better” (objective) way to set a “cut-off “ value or “cut-off” values for which we would either believe H_0 or for which we would not have enough evidence not believe H_0 .

We need to use the normal distribution and probabilities.

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8.5 Hypothesis Test of Mean (σ Known): Classical Approach

THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

Step 2 The Hypothesis Test Criteria:

- a. Check the assumptions.

Assume we know from past experience that $\sigma=4$.

Assume that n is “large” so that by the CLT
 $\bar{x}$ is normally distributed.

$$\mu_{\bar{x}} = \mu, \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad \text{by SDSM}$$

8: Introduction to Statistical Inference

8.5 Hypothesis Test of Mean (σ Known): Classical Approach

THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

Step 2 The Hypothesis Test Criteria:

- b. Identify the probability distribution and the test statistic to be used.

The standard normal distribution is to be used because $\bar{x}$ is expected to have a normal distribution.

Test Statistic for Mean:

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad \text{where } \sigma \text{ is assumed to be known} \quad (8.4)$$

8: Introduction to Statistical Inference

8.5 Hypothesis Test of Mean (σ Known): Classical Approach

THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

Step 2 The Hypothesis Test Criteria:

- c. Determine the level of significance, α .

After much consideration, we assign a tolerable probability of a Type I error to be $\alpha=0.05$.

Type I Error: When a true null hypothesis H_0 is rejected.

8: Introduction to Statistical Inference

8.5 Hypothesis Test of Mean (σ Known): Classical Approach

THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

Step 3 The Sample Evidence:

- a. Collect a sample of information.

Take a random sample from the population with mean μ that being questioned.

- b. Calculate the value of the test statistic.

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{67.2 - 69}{4 / \sqrt{15}} = -1.74$$

Assuming $n=15$ and 67.2 is sample mean. With known $\sigma = 4$.

8: Introduction to Statistical Inference

8.5 Hypothesis Test of Mean (σ Known): Classical Approach

THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

Step 4 The Probability Distribution:

- a. Determine the critical region and critical value(s).

Critical Region: The set of values for the test statistic that will cause us to reject the null hypothesis.

Critical value(s): The “first” or “boundary” value(s) of the critical region(s).

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8.5 Hypothesis Test of Mean (σ Known): Classical Approach

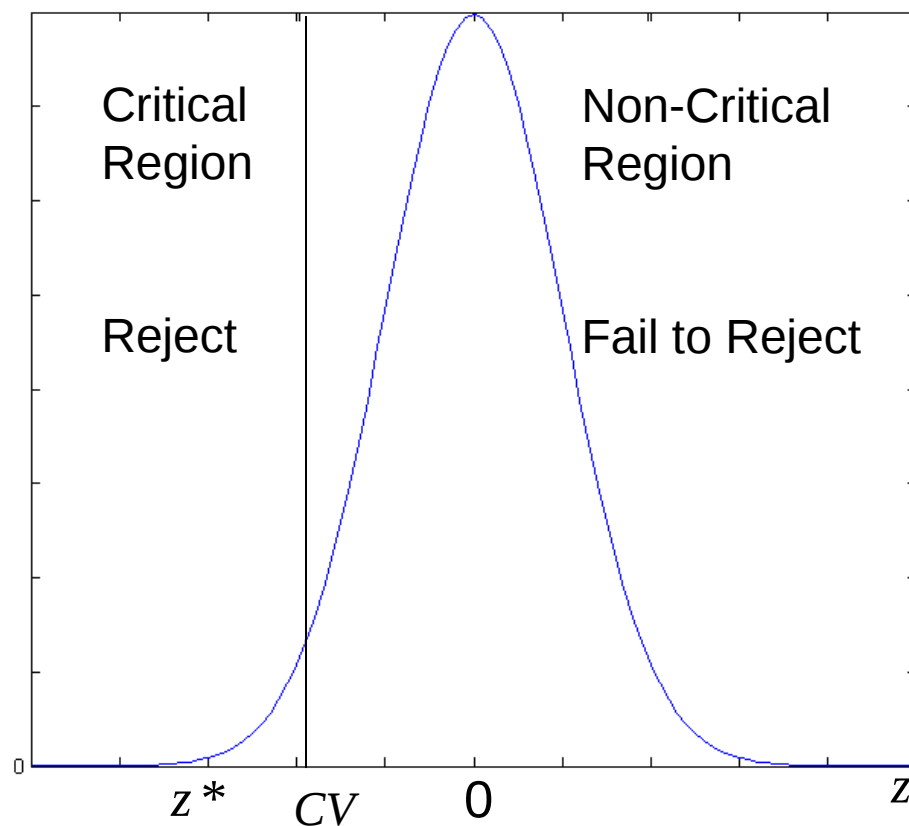
There are three possible hypothesis pairs for the mean.

$$H_0: \mu \geq \mu_0 \text{ vs. } H_a: \mu < \mu_0$$

Reject H_0 if less than

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad -z(\alpha)$$

data indicates $\mu < \mu_0$
because $\bar{x}$ is “a lot”
smaller than μ_0



8: Introduction to Statistical Inference

8.5 Hypothesis Test of Mean (σ Known): Classical Approach

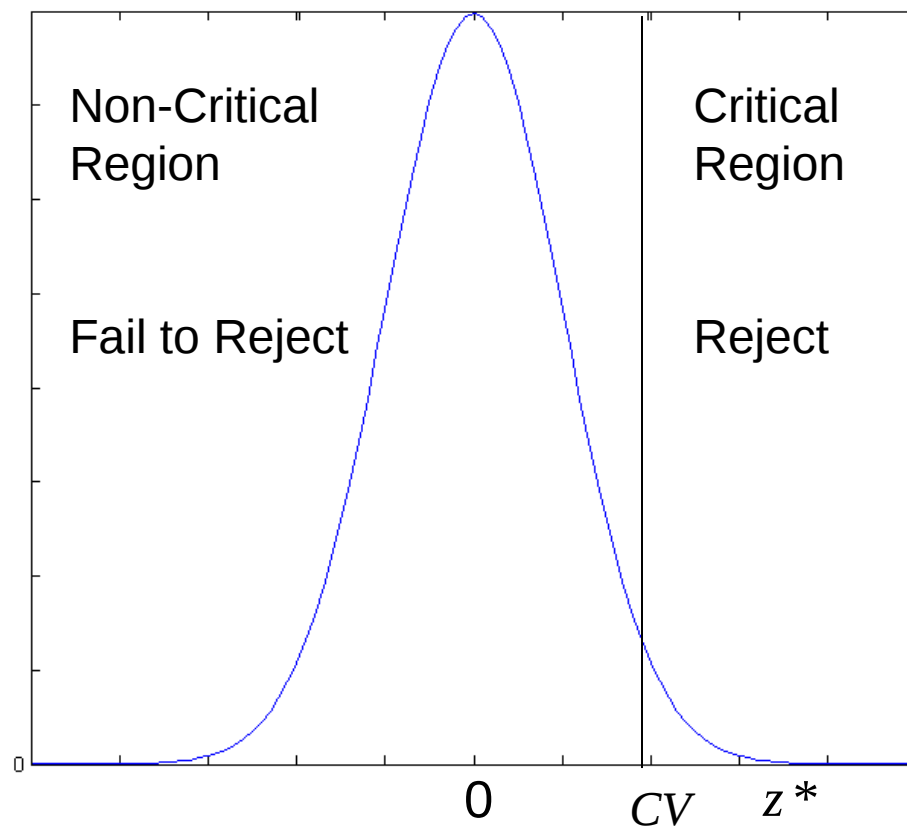
There are three possible hypothesis pairs for the mean.

$$H_0: \mu \leq \mu_0 \text{ vs. } H_a: \mu > \mu_0$$

Reject H_0 if z is greater than

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad z(\alpha)$$

data indicates $\mu > \mu_0$
because $\bar{x}$ is “a lot”
larger than μ_0



8: Introduction to Statistical Inference

8.5 Hypothesis Test of Mean (σ Known): Classical Approach

There are three possible hypothesis pairs for the mean.

$$H_0: \mu = \mu_0 \text{ vs. } H_a: \mu \neq \mu_0$$

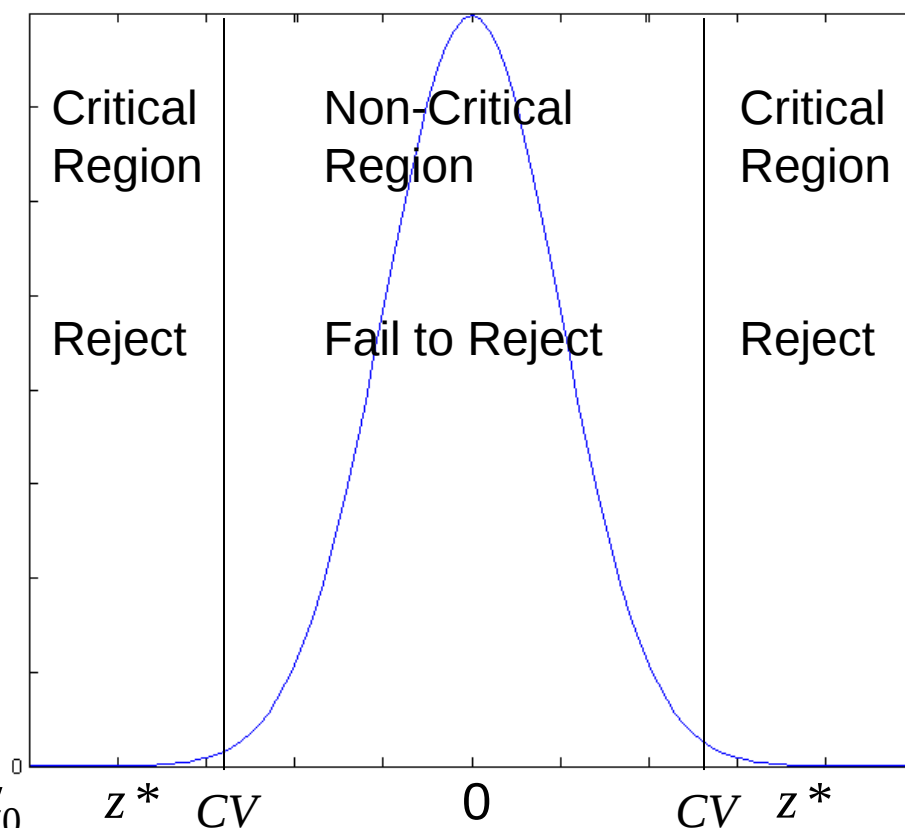
Reject H_0 if less than

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad -z(\alpha / 2)$$

or if greater than

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad z(\alpha / 2)$$

data indicates $\mu \neq \mu_0$, $\bar{x}$ far from μ_0



8: Introduction to Statistical Inference

8.5 Hypothesis Test of Mean (σ Known): Classical Approach

THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

Step 4 The Probability Distribution:

- Determine the critical region and critical value(s).
- Determine whether or not the calculated test statistic is in the critical region.

critical region	noncritical region	critical region
-----------------	--------------------	-----------------

$$H_0: \mu = 69'' \text{ vs. } H_a: \mu \neq 69''$$

$$P(z > z(\alpha / 2)) = \alpha / 2 ,$$

$$z(.025) = 1.96$$

since two sided test.

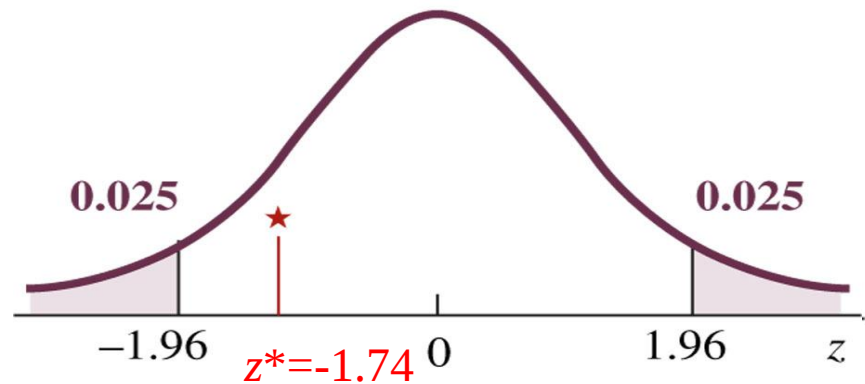


Figure from Johnson & Kubly, 2012.

8: Introduction to Statistical Inference

8.5 Hypothesis Test of Mean (σ Known): Classical Approach

THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

Step 5 The Results:

- a. State the decision about H_0 .
Need a decision rule.

Decision rule:

- a. If the test statistic falls *within the critical region*, then the decision must be **reject H_0** .
- b. If the test statistic is *not in the critical region*, then the decision must be **fail to reject H_0** .

8: Introduction to Statistical Inference

8.5 Hypothesis Test of Mean (σ Known): Classical Approach

THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

Step 5 The Results:

b. State the conclusion about H_a .

With $\alpha = 0.05$,

there is not sufficient evidence to reject H_0 .

Fail to reject H_0 .

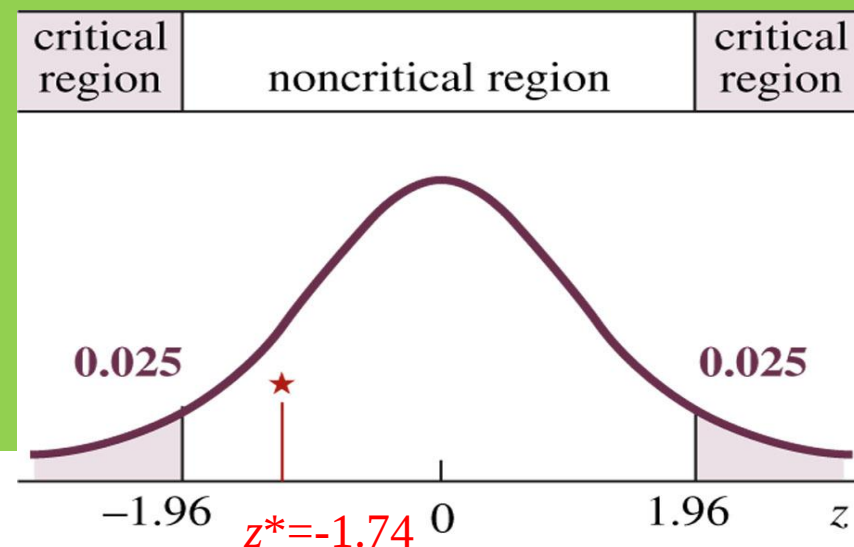


Figure from Johnson & Kuby, 2012.

8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

8.5 Hypothesis Test of Mean (σ Known): Classical Approach

Let's examine the hypothesis test

$$H_0: \mu \leq 69'' \text{ vs. } H_a: \mu > 69''$$

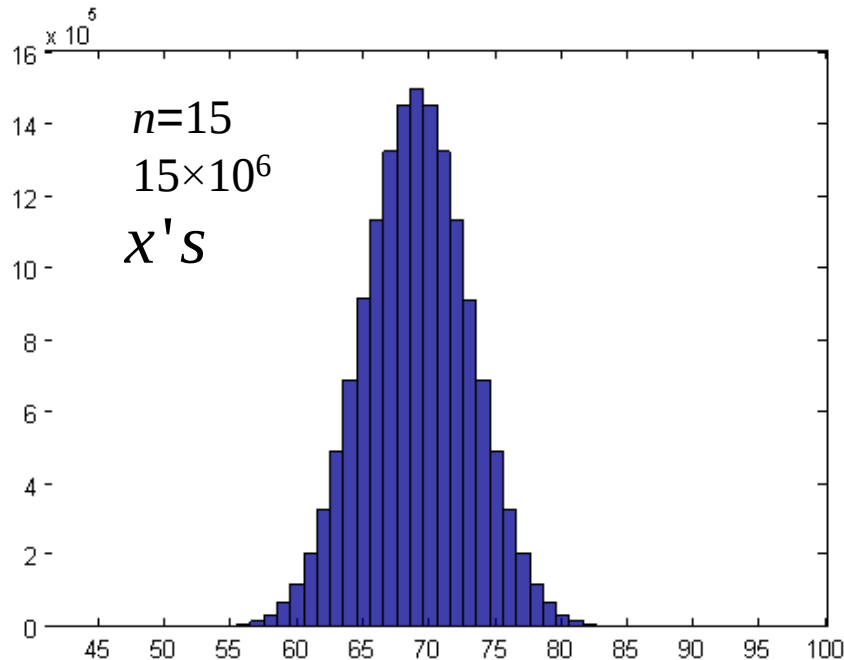
with $\alpha=0.05$ for the heights of Math 1700 students.

Generate random data values.

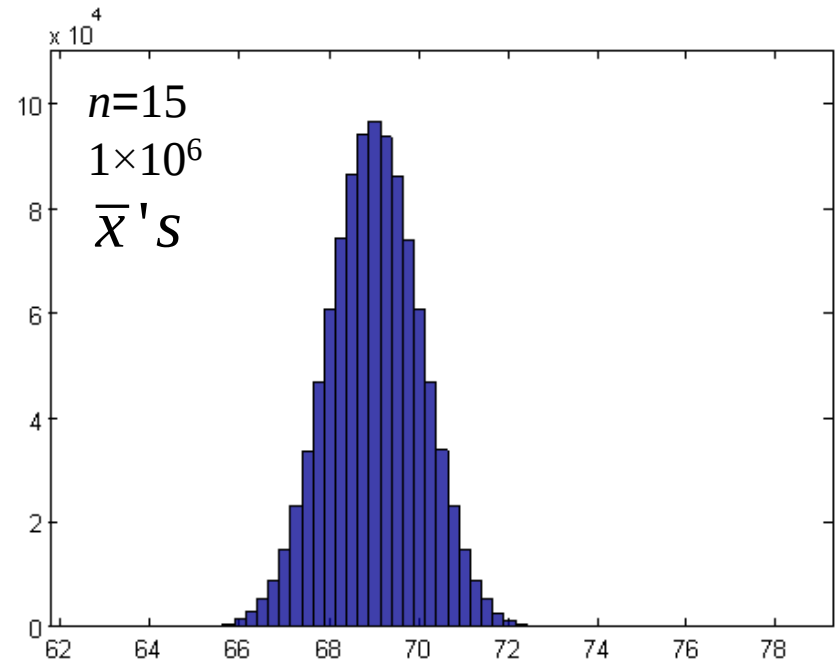
8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

8.5 Hypothesis Test of Mean (σ Known): Classical Approach

Generated 15×10^6
normal data values
from $\mu = 69$ " and $\sigma = 4$ ".

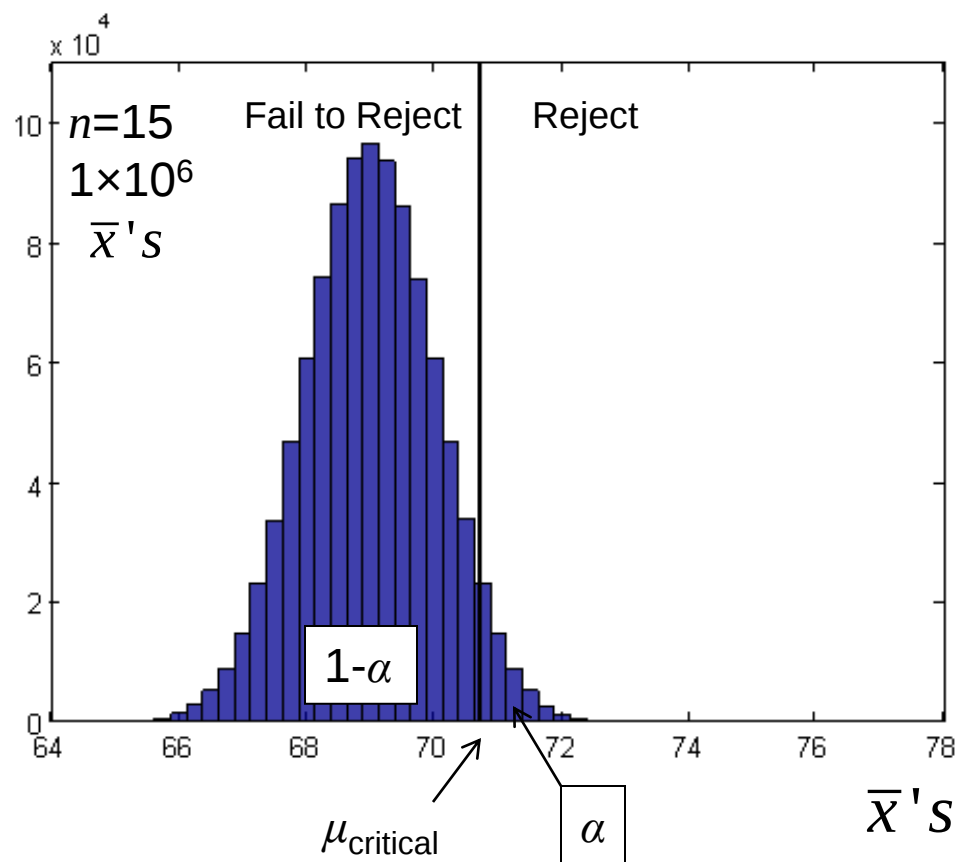


Calculated
 1×10^6 means with $n=15$.
(Will repeat for $\mu = 72$ ".)



8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

8.5 Hypothesis Test of Mean (σ Known): Classical Approach



$$H_0: \mu \leq 69'' \text{ vs. } H_a: \mu > 69''$$

$$\alpha = .05$$

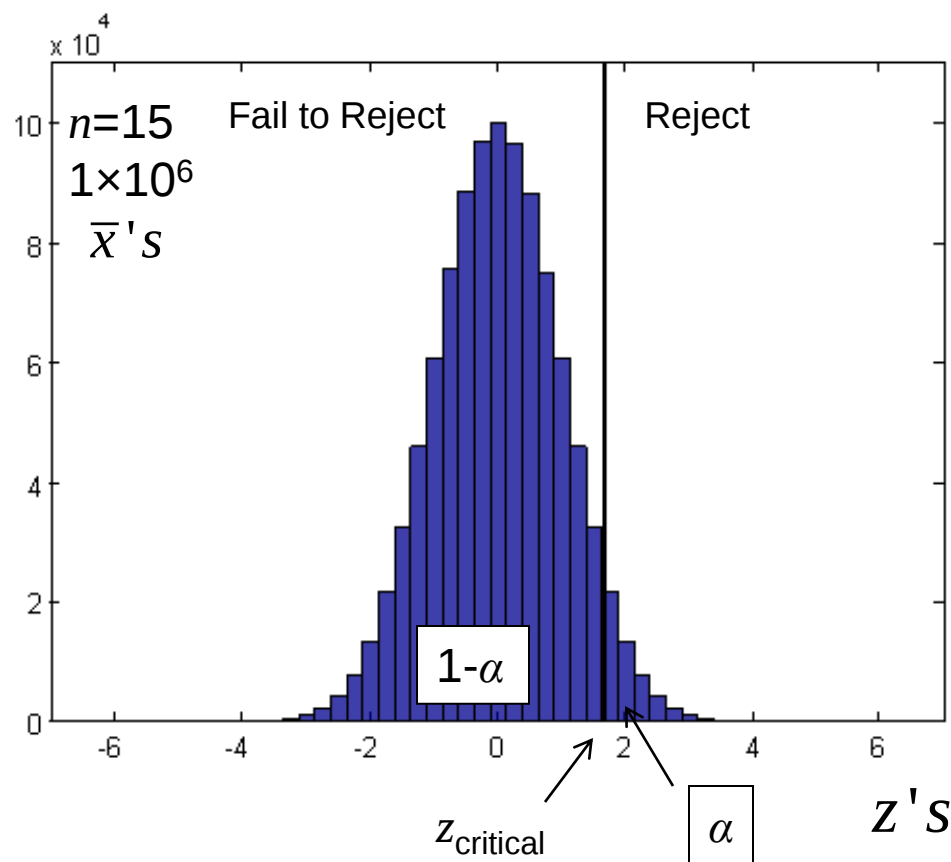
When the true mean $\mu = 69''$, we reject H_0 α fraction of the time.

Commit a Type I Error.

Given α , we want $\mu_{critical}$.

8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

8.5 Hypothesis Test of Mean (σ Known): Classical Approach



Instead of $\mu_{critical}$ we find critical z , $z_{critical} = z(\alpha)$.

Do this by assuming that $H_0: \mu = 69$ is true, then calculate

$$z = \frac{\bar{x} - 69}{4 / \sqrt{15}}$$

8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

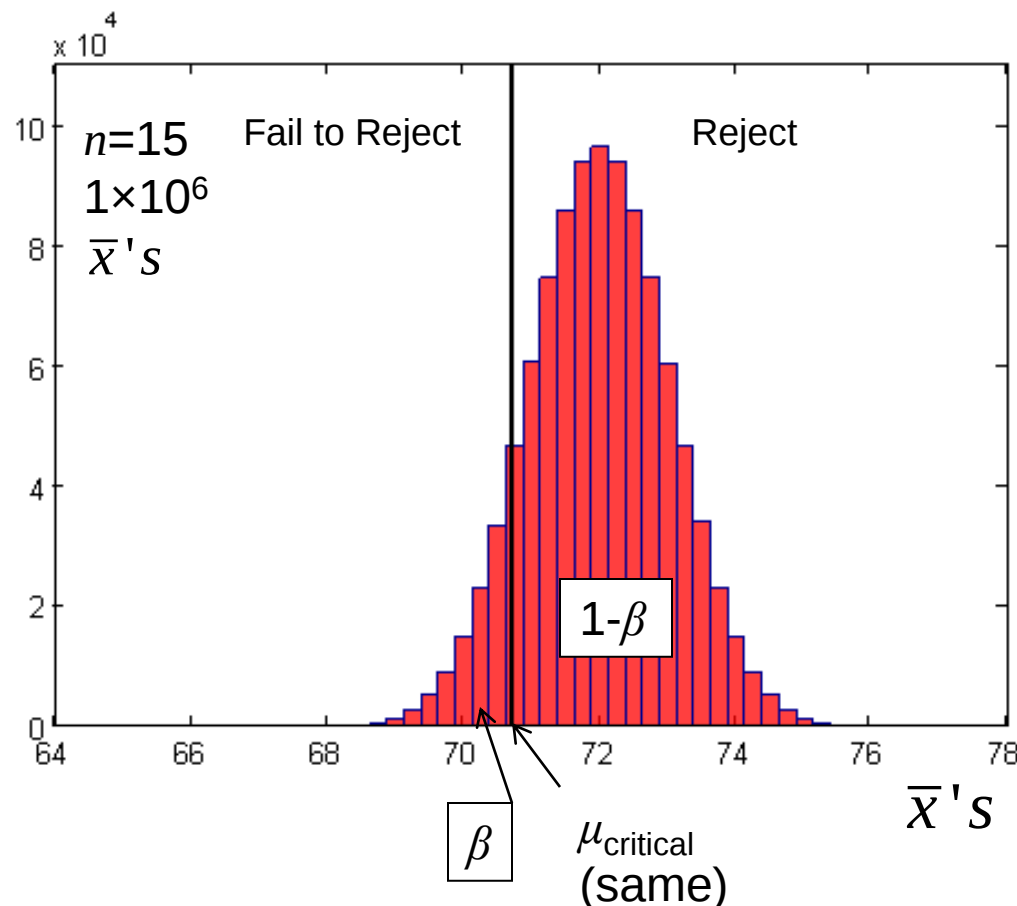
8.5 Hypothesis Test of Mean (σ Known): Classical Approach

$$H_0: \mu \leq 69 \text{ vs. } H_a: \mu > 69$$

$$\alpha = .05$$

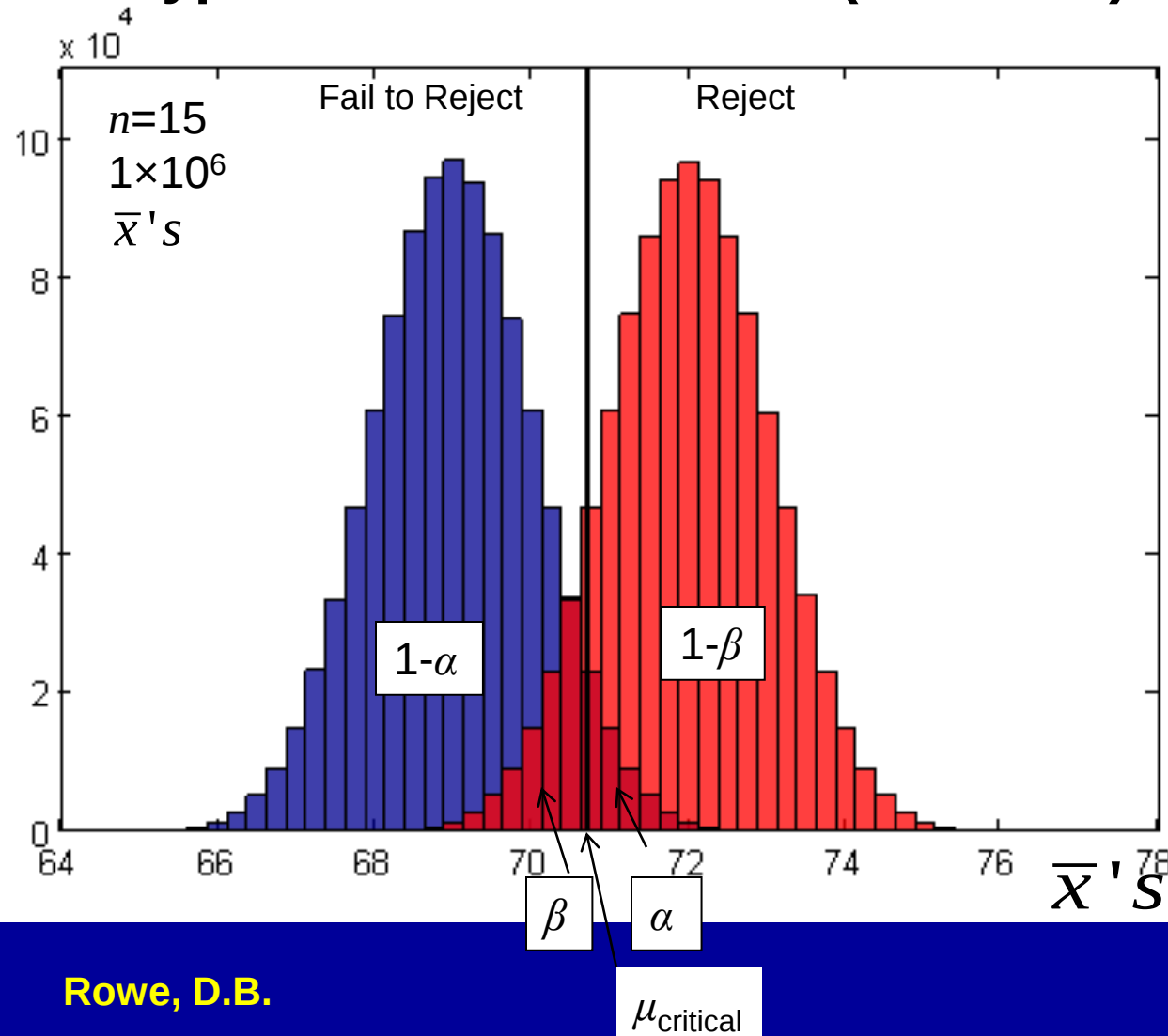
When the true mean $\mu = 72$ ",
we do not reject H_0 β
fraction of the time.

Commit a Type II Error



8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

8.5 Hypothesis Test of Mean (σ Known): Classical Approach



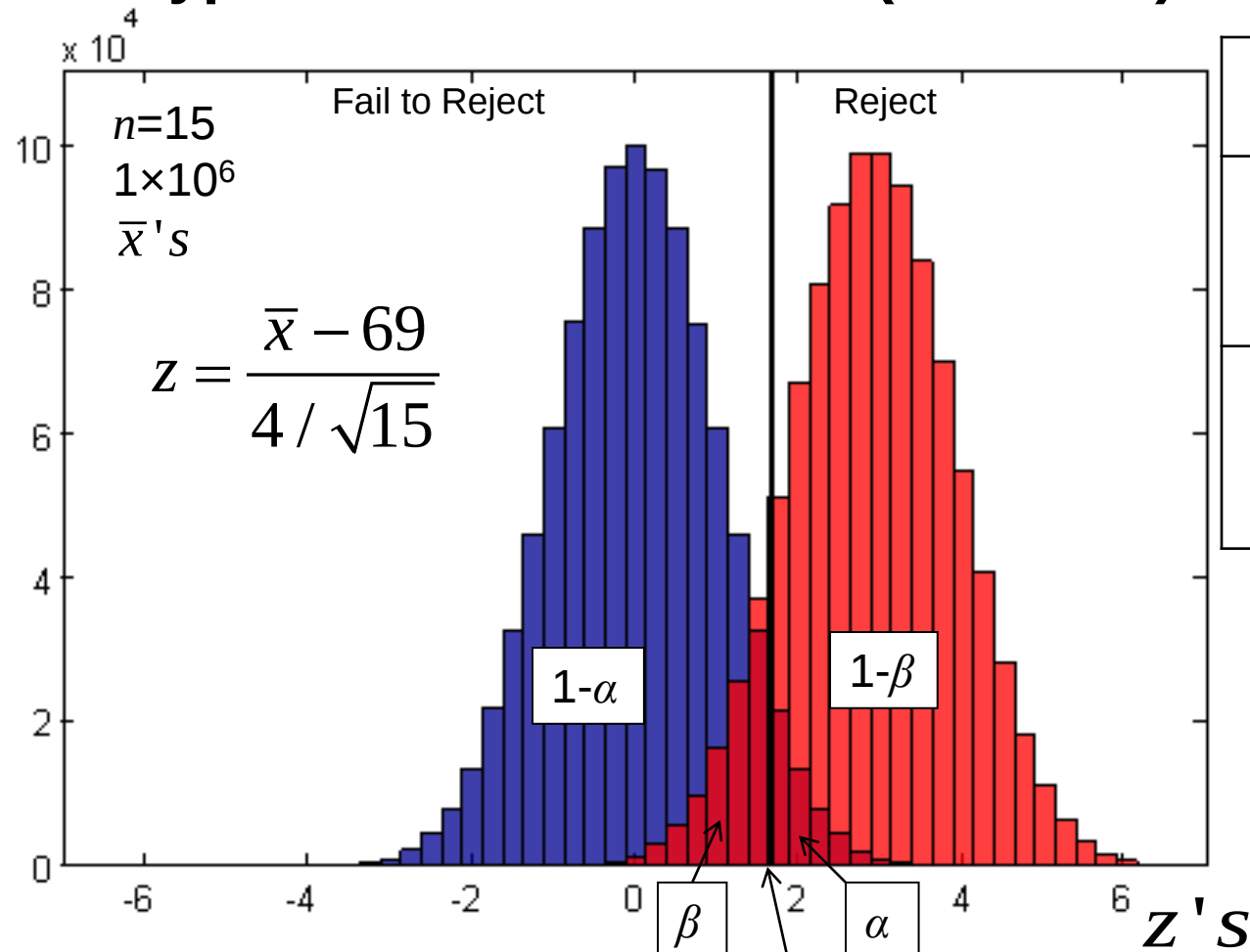
	H_0 True ($\mu=69''$)	H_0 False ($\mu=72''$)
Fail to Reject H_0	Correct Decision ($1-\alpha$)	Type II Error (β)
Reject H_0	Type I Error (α)	Correct Decision ($1-\beta$)

Power of the test:
 $1 - \beta = P(\text{Reject } H_0 \mid H_0 \text{ False})$

Discrimination ability.
 Ability to detect difference.

8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

8.5 Hypothesis Test of Mean (σ Known): Classical Approach



	H_0 True ($\mu=69''$)	H_0 False ($\mu=72''$)
Fail to Reject H_0	Correct Decision ($1-\alpha$)	Type II Error (β)
Reject H_0	Type I Error (α)	Correct Decision ($1-\beta$)

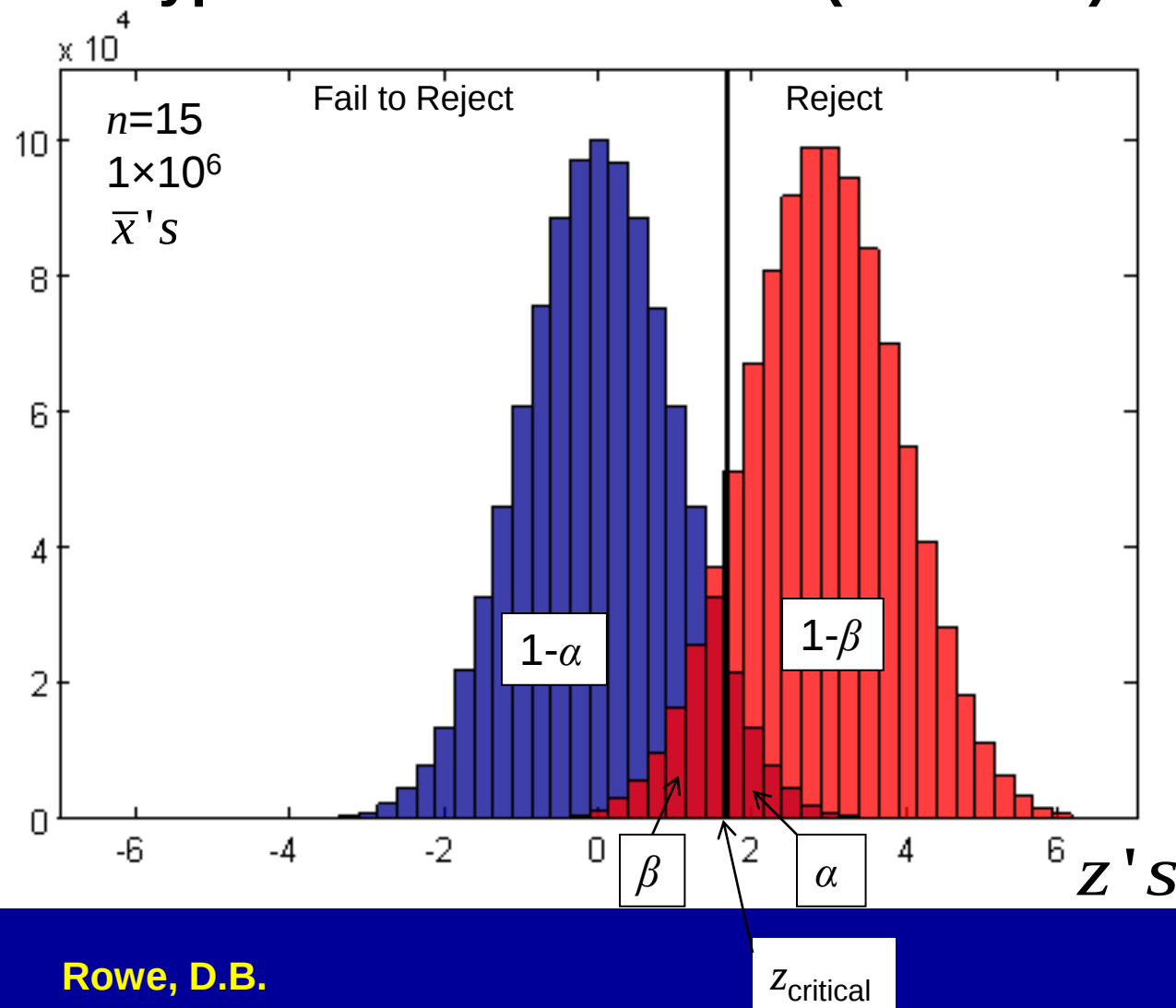
Power of the test:

$$1 - \beta = P(\text{Reject } H_0 \mid H_0 \text{ False})$$

Discrimination ability.
Ability to detect difference.

8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

8.5 Hypothesis Test of Mean (σ Known): Classical Approach



We want our α ,
Prob of Type I Error
to be small.

So why not just
decrease α ?

Decreasing α
increases β .

And vice versa.

8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

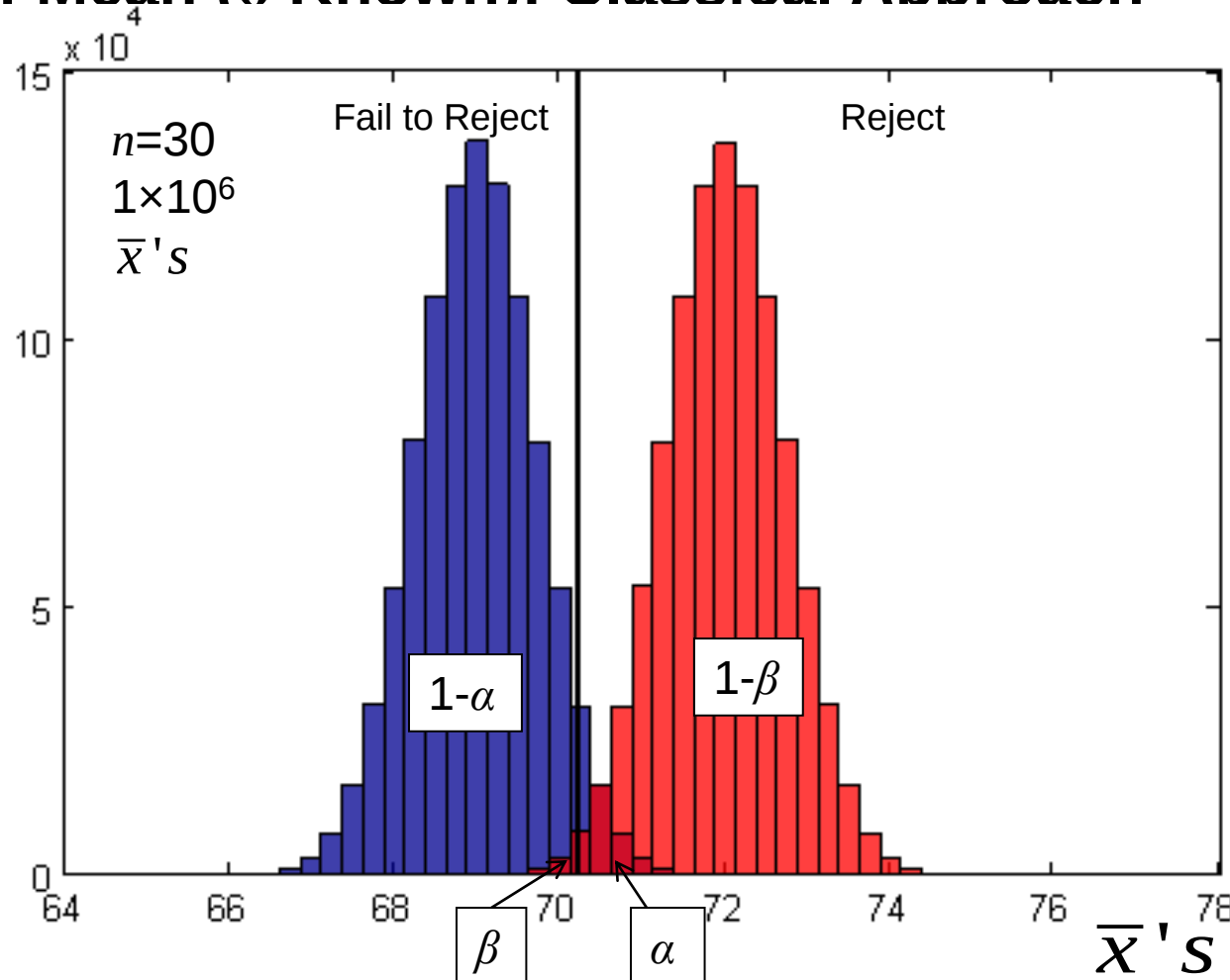
8.5 Hypothesis Test of Mean (σ Known): Classical Approach

What is the solution?

Increase n .

Figure shows n increased to $n=30$ from $n=15$.

Note α and β both smaller with larger n .



8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

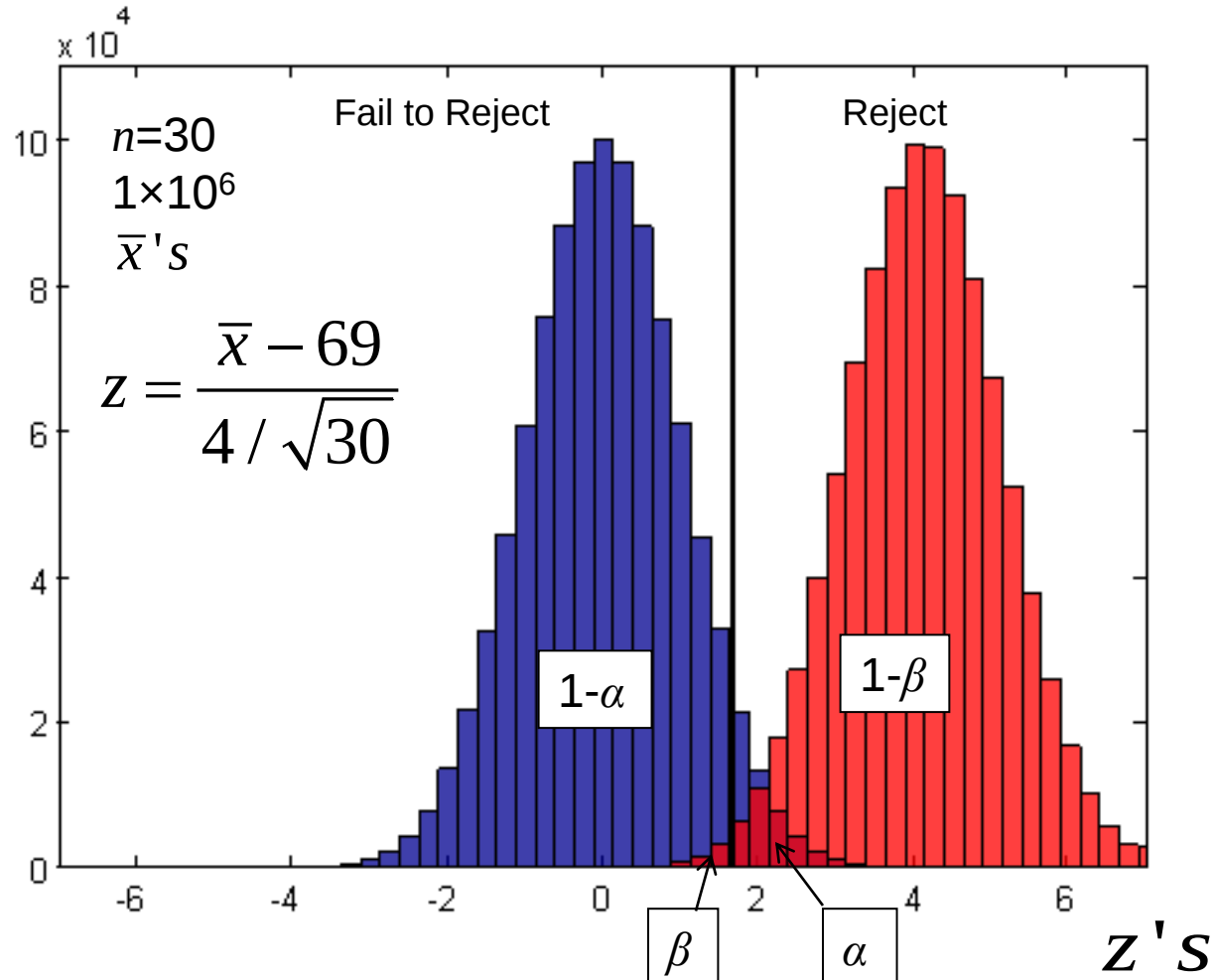
8.5 Hypothesis Test of Mean (σ Known): Classical Approach

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Chapter 8: Introduction to Statistical Inference

Questions?

Homework: Read 8.3-8.5

WebAssign

Chapter 8 # 57, 59, 81, 93, 97, 106, 109,
119, 145, 157