# Class 10

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# Lecture Chapter 6.1- 6.5

# Return Exams?

# Chapter 6: Normal Probability Distributions (Continuous Distribution)

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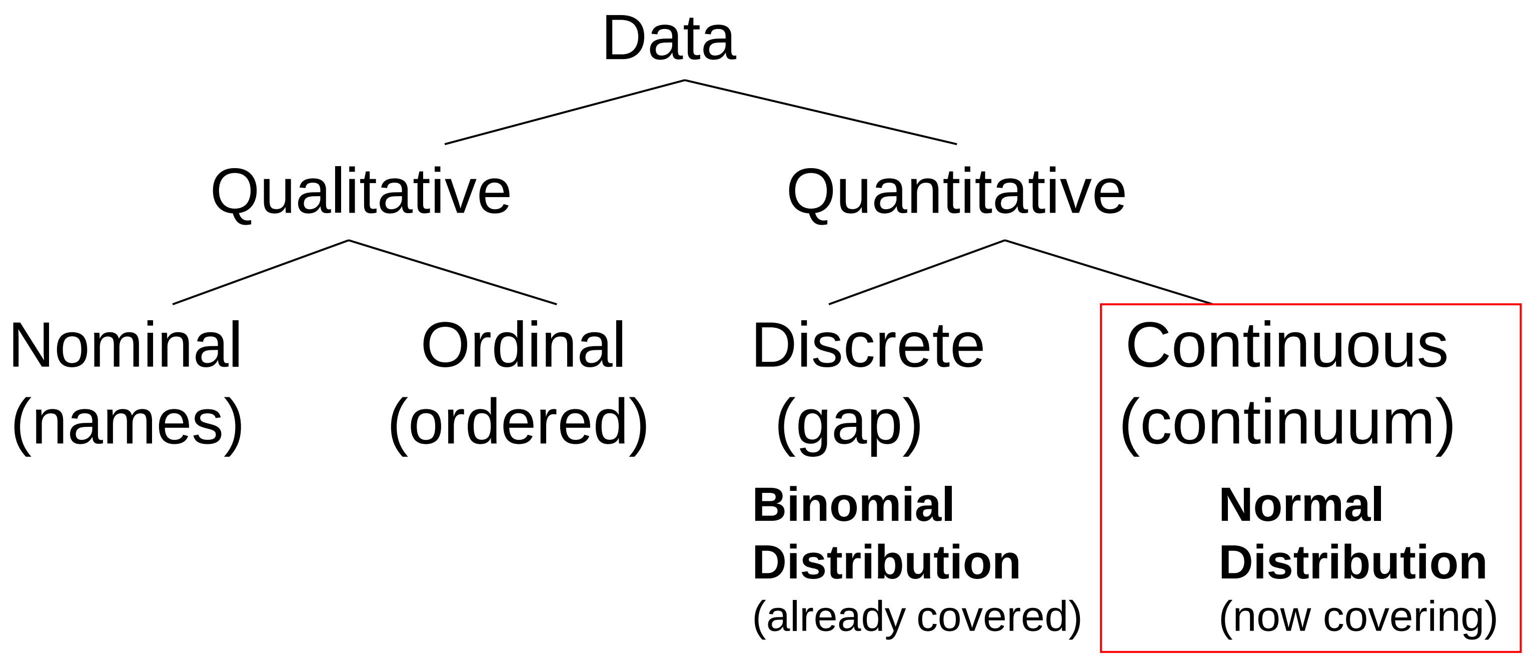
MARQUETTE UNIVERSITY
Be The Difference.

# 6: Normal Probability Distributions

## 6.1 Normal Probability Distributions

At the beginning of course we talked about types of data.



# 6: Normal Probability Distributions

## 6.1 Normal Probability Distributions

We discussed discrete random variables and discrete probability functions, $P(x)$.

**Probability Function:** A rule $P(x)$ that assigns probabilities to the values of the random variables, $x$.

**Example:**

Let $x$ = # of heads when we flip a coin twice.

$x=\{0,1,2\}$

$$P(x)=\frac{2!}{x!(2-x)!}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{2-x}$$

# 6: Normal Probability Distributions

## 6.1 Normal Probability Distributions

The most important continuous distribution is the normal distribution (p 269). Insert $x$ and get $f(x)$.

**Probability distribution, continuous variable:** … the probability for a continuous random variable, $x$, having values falling within a specified interval.

**Normal Probability Distribution Function:**

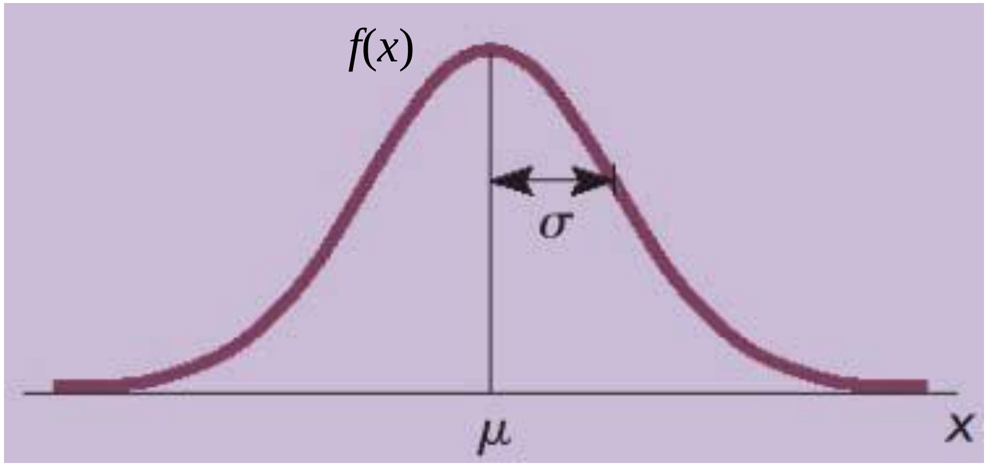
$$y = f(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}} \quad \text{for all } x \text{ real} \tag{6.1}$$

# 6: Normal Probability Distributions

## 6.1 Normal Probability Distributions

The mathematical formula for the normal distribution is (p 269):

$$f(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$




where

$e$ = 2.718281828459046…

$\pi$ = 3.141592653589793…

$\mu$ = population mean

$\sigma$ = population std. deviation

$$-\infty < x, \mu < +\infty$$
$$0 < \sigma$$

We will not use this formula.

Figure from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions
## 6.1 Normal Probability Distributions

**Properties of Normal Distribution**

1. Total Area under the normal curve is 1
2. Mound shaped, symmetric about mean, extends to $\pm\infty$
3. Has a mean of $\mu$ and standard deviation $\sigma$.
4. The mean divides area in half.
5. Nearly all area within 3$\sigma$ of $\mu$.

$$f(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

$$-\infty < x, \mu < +\infty \qquad 0 < \sigma$$

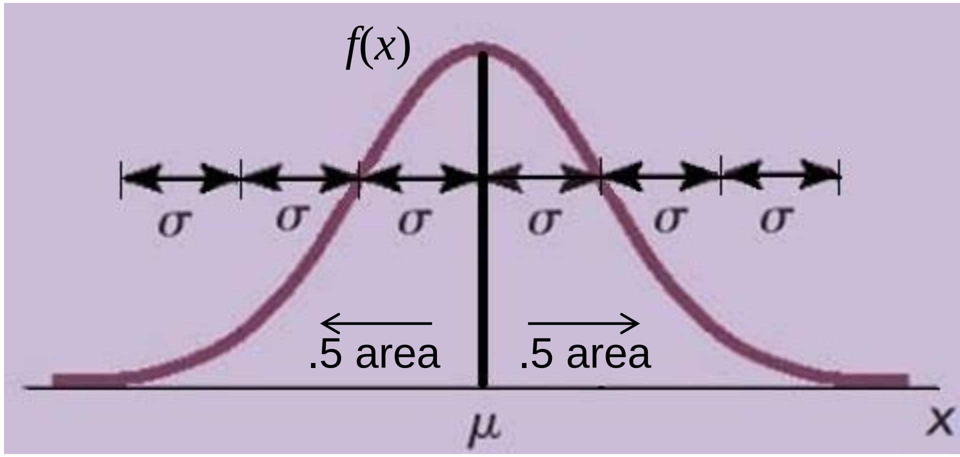



Figure modified from Johnson & Kuby, 2012.

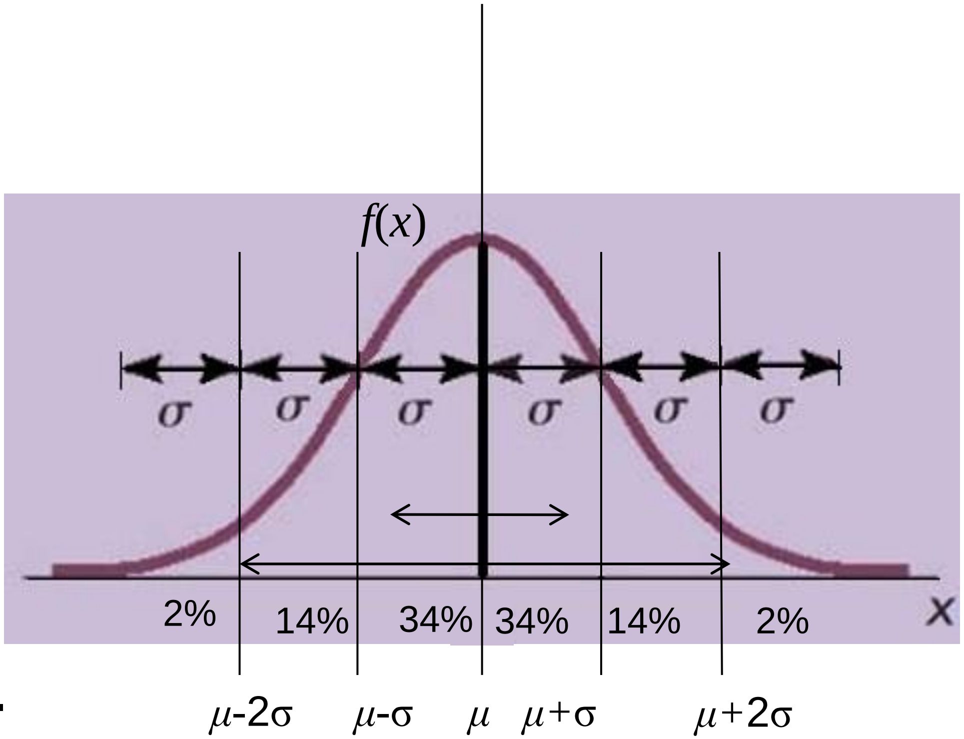
# 6: Normal Probability Distributions

## 6.1 Normal Probability Distributions

1. Symmetric about the mean.
2. mean = median = mode.
3. Mean $\mu$ & variance $\sigma^2$ completely characterize.
4. $P(\mu-\sigma<x<\mu+\sigma)=.68$

   $P(\mu-2\sigma<x<\mu+2\sigma)=.95$

   $P(\mu-3\sigma<x<\mu+3\sigma)=.99$ .




5. $P(a<x<b)=$ area under curve from $a$ to $b$ .

Figure modified from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions

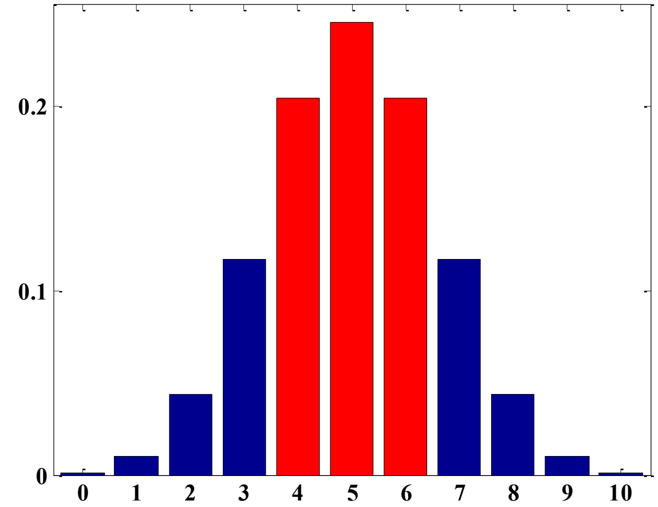
## 6.1 Normal Probability Distributions

When we discussed random experiments such as flipping a coin or rolling a die, we described the outcomes and events.

We then discussed the probabilities of these events which consisted of probabilities of the individual outcomes.

With the discrete binomial distribution we were interested in events such as

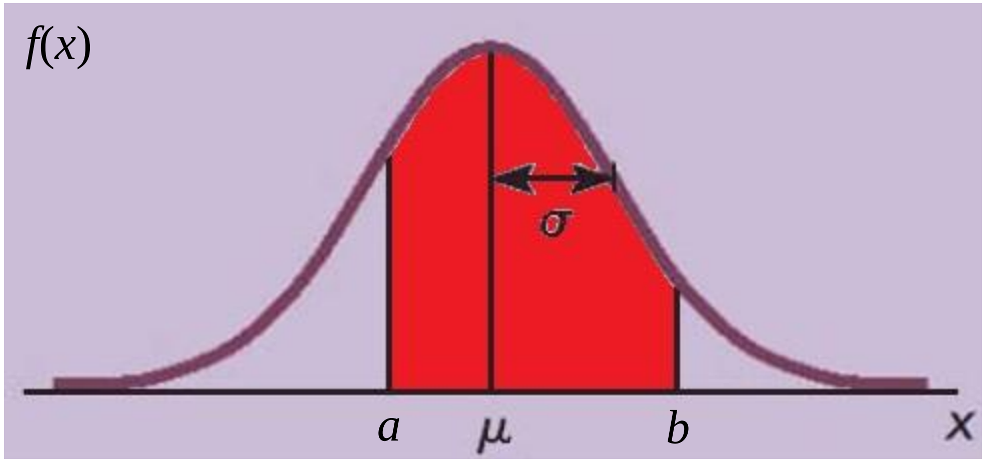
$P(4 \le x \le 6) = P(4) + P(5) + P(6)$



# 6: Normal Probability Distributions

## 6.1 Normal Probability Distributions

With the continuous normal distribution, we want areas.



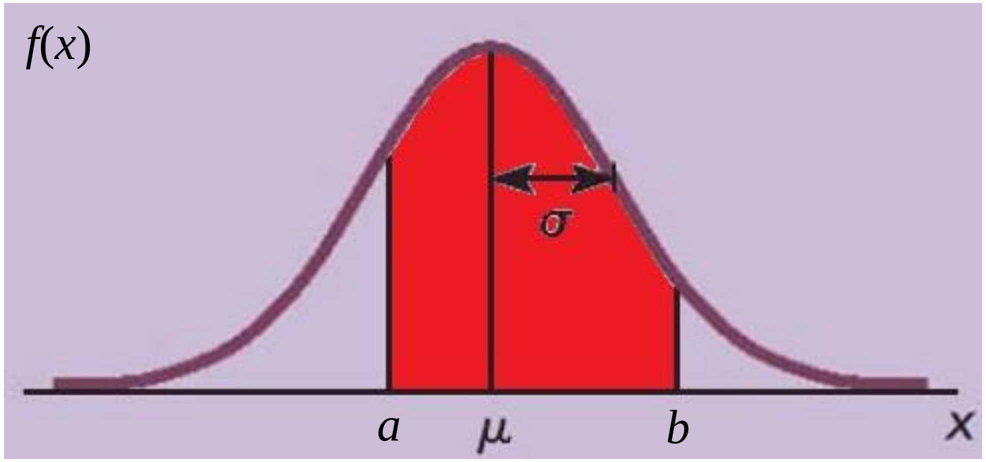

The probability $x$ is in the interval $a$ to $b$ is in red

Shaded area:
$P(a \leq x \leq b)$

Figure modified from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions
## 6.1 Normal Probability Distributions

Areas of continuous functions are found with Calculus.




Aside: Don't need to know.

$$A = \int_a^b \underbrace{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}_{f(x)} dx \quad (6.2)$$

This can not be done with pencil & paper and can only be done numerically with a computer.

We will not use Calculus in this class.

Figure modified from Johnson & Kuby, 2012.

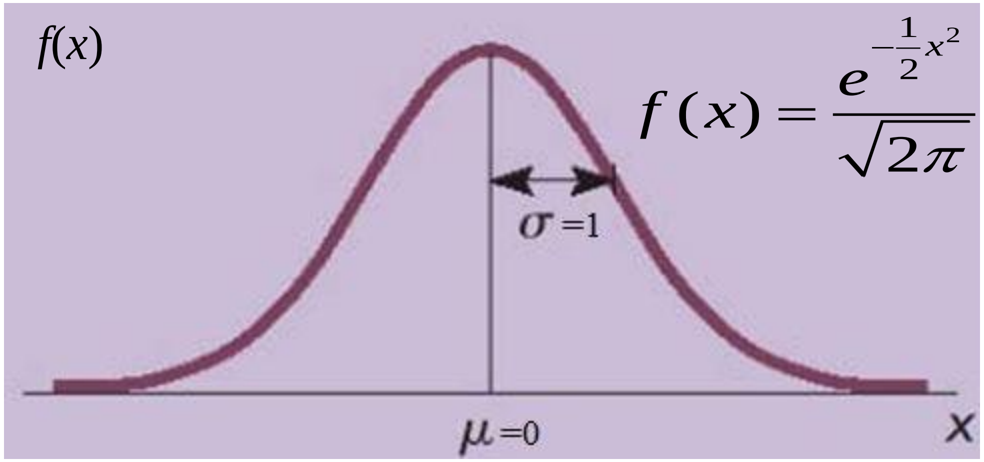
# 6: Normal Probability Distributions

## 6.1 Normal Probability Distributions

How are we going to find areas in this class?

We find areas of the normal distribution by using the standard normal distribution and tables in the back of the book.

When $\mu$=0 and σ=1, the curve is called the "standard" normal distribution.




I will describe the standard normal, then discuss finding areas.

Figure from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions

## 6.2 The Standard Normal Distribution

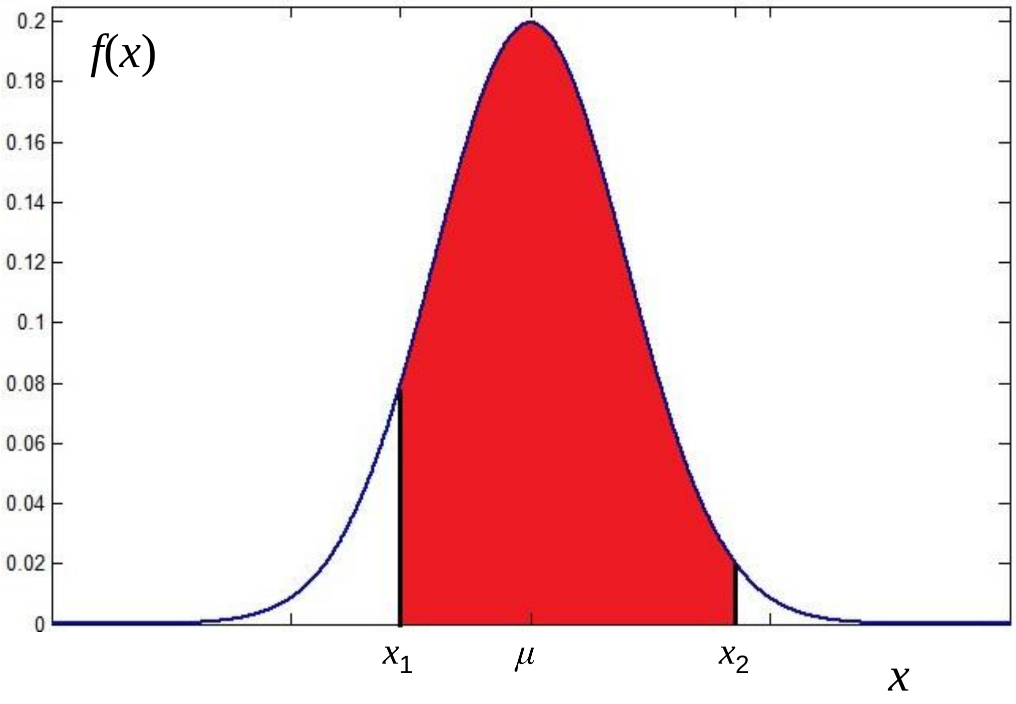
Properties of the Standard Normal Distribution:

1. Total area under the **normal curve** is 1.
2. The distribution is mounded and symmetric, it extends indefinitely in both directions; approaching but never touching the horizontal axis.
3. The distribution has a mean of 0 and a standard deviation of 1.
4. The mean divides the area in half, .5 on each side.
5. Nearly all the area is between $z$ =-3.00 and $z$=3.00.

# 6: Normal Probability Distributions

## 6.2 The Standard Normal Probability Distributions

Normal distribution with population mean $\mu$ and variance $\sigma^2$.




We want to know the (red) area under the normal distribution between $x_1$ and $x_2$.

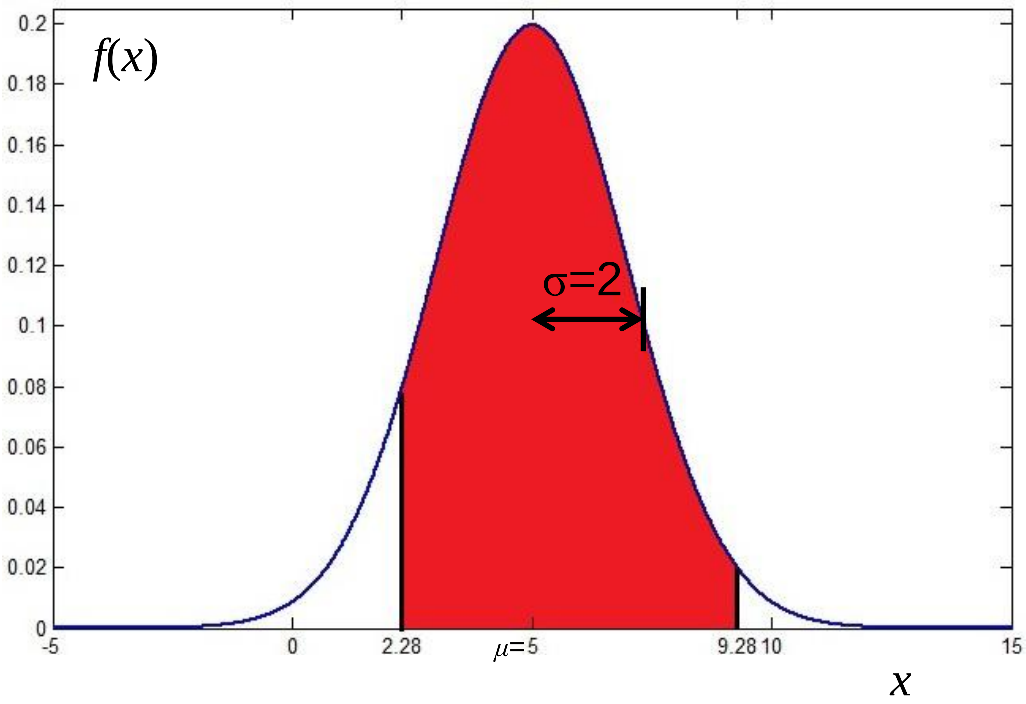
Note:
Similar to discrete probabilities adding to 1.

The total area under the normal distribution is 1.

# 6: Normal Probability Distributions

## 6.2 The Standard Normal Probability Distributions

**Example:** Here is a normal distribution with $\mu = 5$ and $\sigma^2 = 4$.



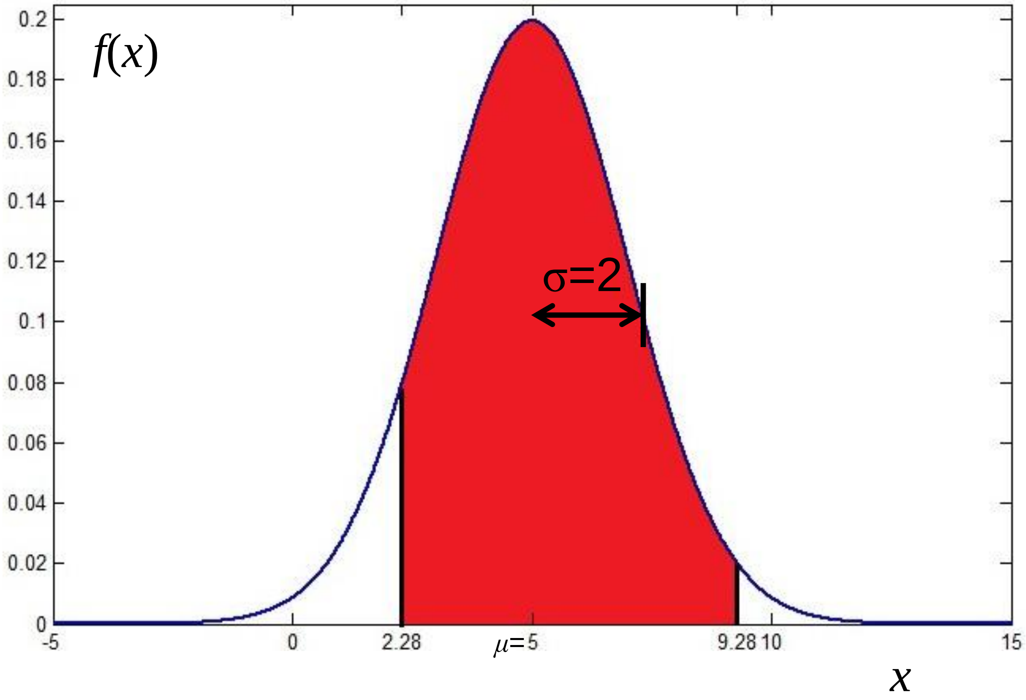

Let's say we want to know the red area under the normal distribution between $x_1 = 2.28$ and $x_2 = 9.28$.

What is the area under the normal distribution between these two values?

# 6: Normal Probability Distributions

## 6.2 The Standard Normal Probability Distributions

**Example:** Here is a normal distribution with $\mu = 5$ and $\sigma^2 = 4$.



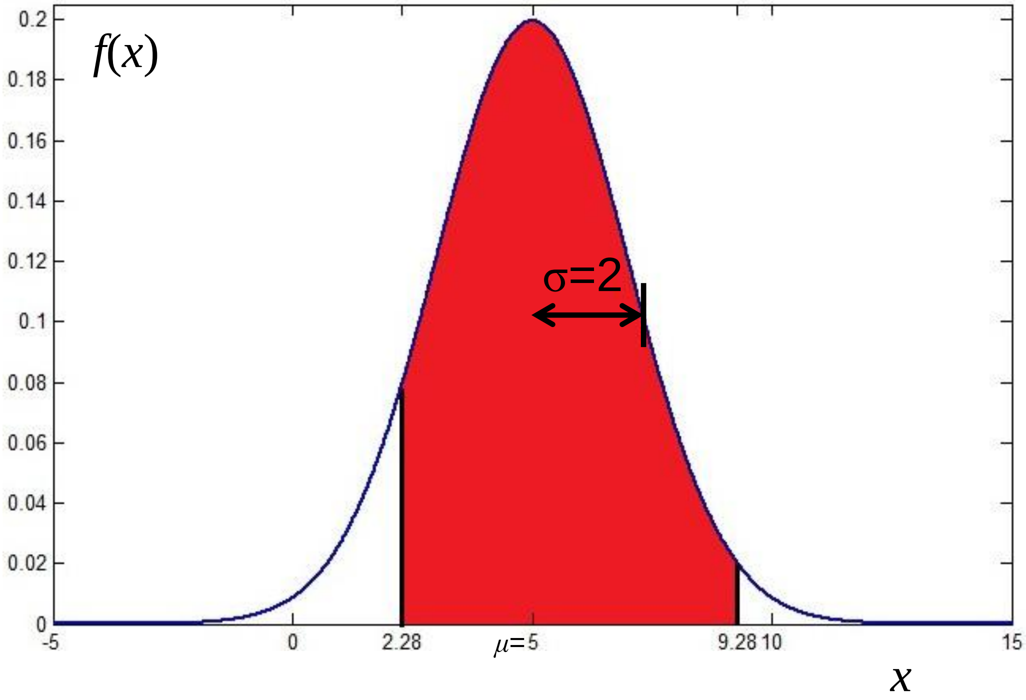

Aside: Don't need to know.

$$A = \int_{2.28}^{9.28} \underbrace{\frac{1}{2\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-5}{2}\right)^2}}_{f(x)} dx$$

We would normally do this numerically with a computer.

# 6: Normal Probability Distributions
## 6.2 The Standard Normal Probability Distributions

**Example:** Here is a normal distribution with $\mu = 5$ and $\sigma^2 = 4$.




But we can't do calculus in this class.

Someone had the idea to convert normal distribution to the "standard" normal.

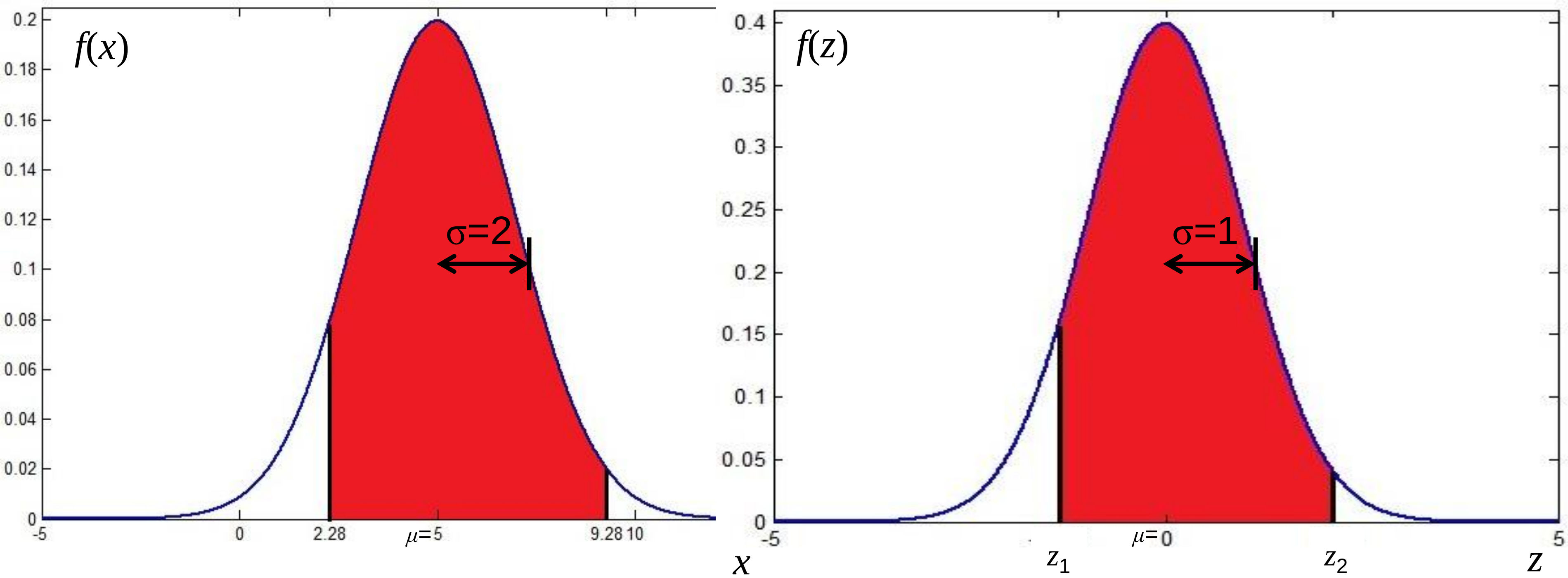
Subtract $\mu$ and divide this by $\sigma$ for every value of $x$.
$z = (x- \mu)/\sigma$.

Area between $x_1$ and $x_2$ is the same as area between $z_1$ and $z_2$.

# 6: Normal Probability Distributions

## 6.2 The Standard Normal Probability Distributions

**Example:** Here is a normal distribution with $\mu = 5$ and $\sigma^2 = 4$.

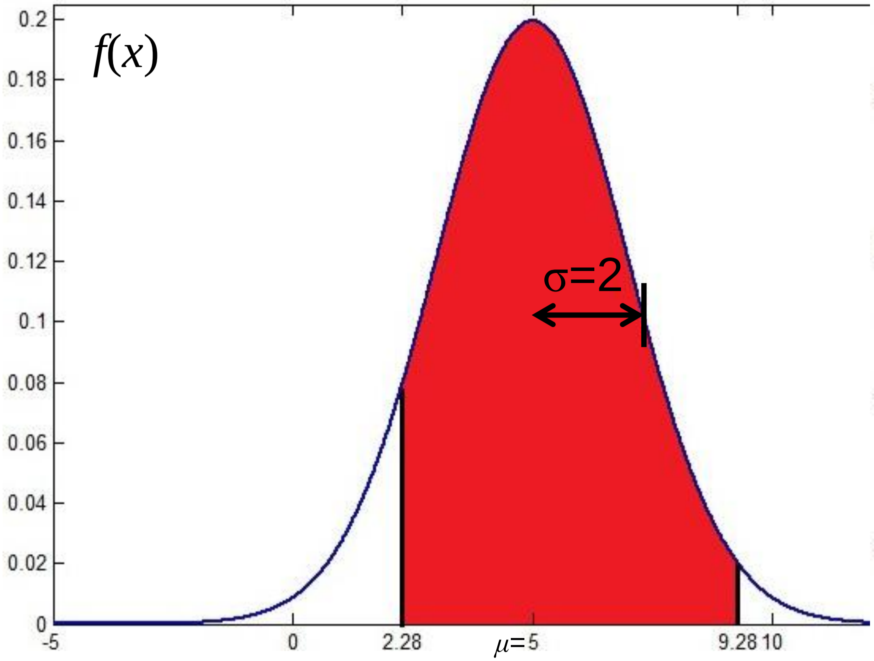


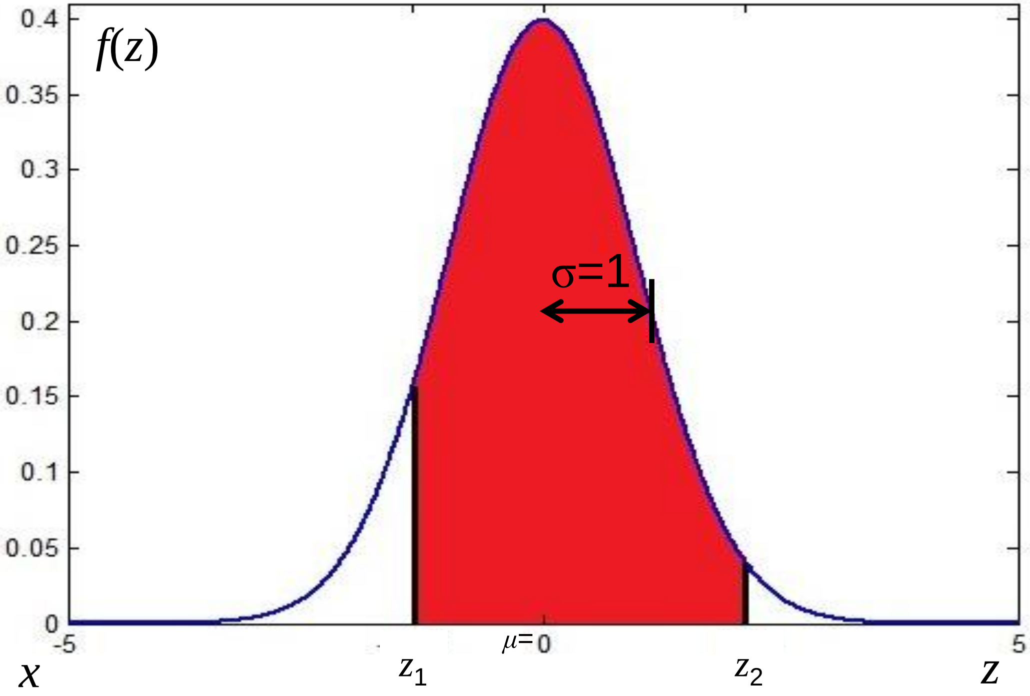

Area between $x_1$ and $x_2$ is the same as area between $z_1$ and $z_2$.

# 6: Normal Probability Distributions

## 6.2 The Standard Normal Probability Distributions

**Example:** Here is a normal distribution with $\mu = 5$ and $\sigma^2 = 4$.



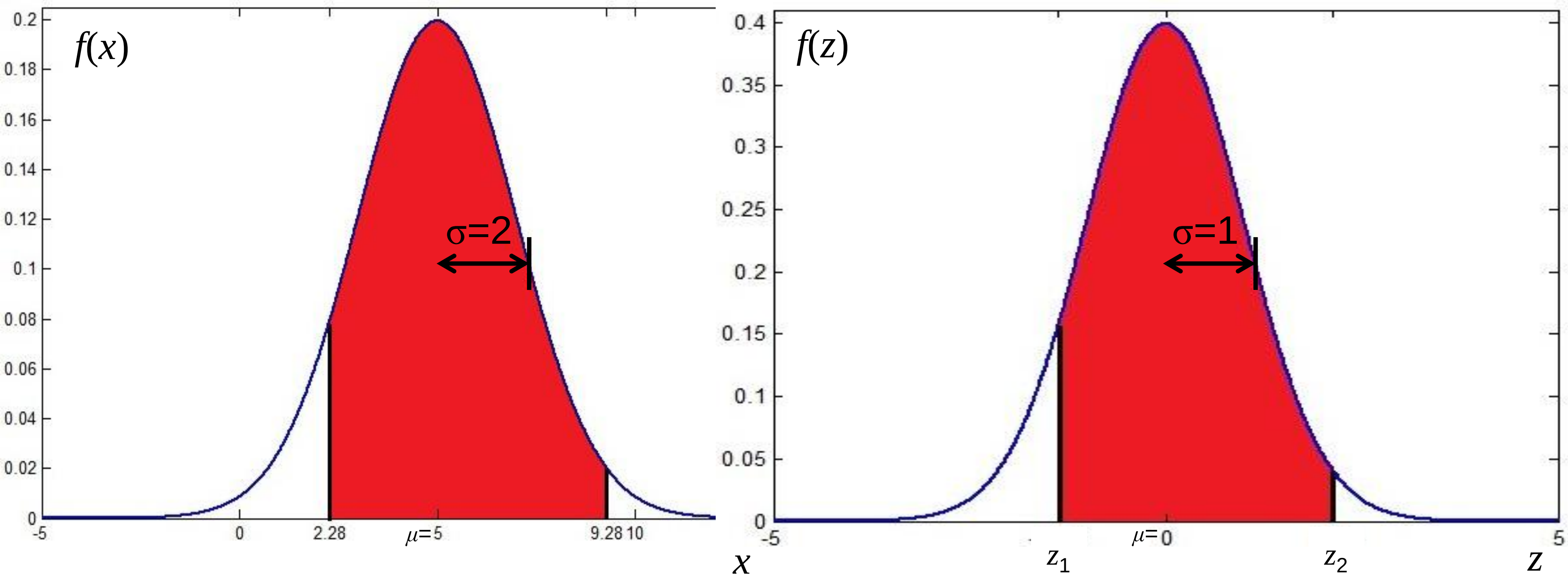




If $x_1 = 2.28$ and $x_2 = 9.28$ then $z_1 = (x_1 - \mu)/\sigma$ and $z_2 = (x_2 - \mu)/\sigma$ are?

# 6: Normal Probability Distributions

## 6.2 The Standard Normal Probability Distributions

**Example:** Here is a normal distribution with $\mu = 5$ and $\sigma^2 = 4$.




If $x_1 = 2.28$ and $x_2 = 9.28$ then $z_1 = (x_1 - \mu)/\sigma$ and $z_2 = (x_2 - \mu)/\sigma$ are?

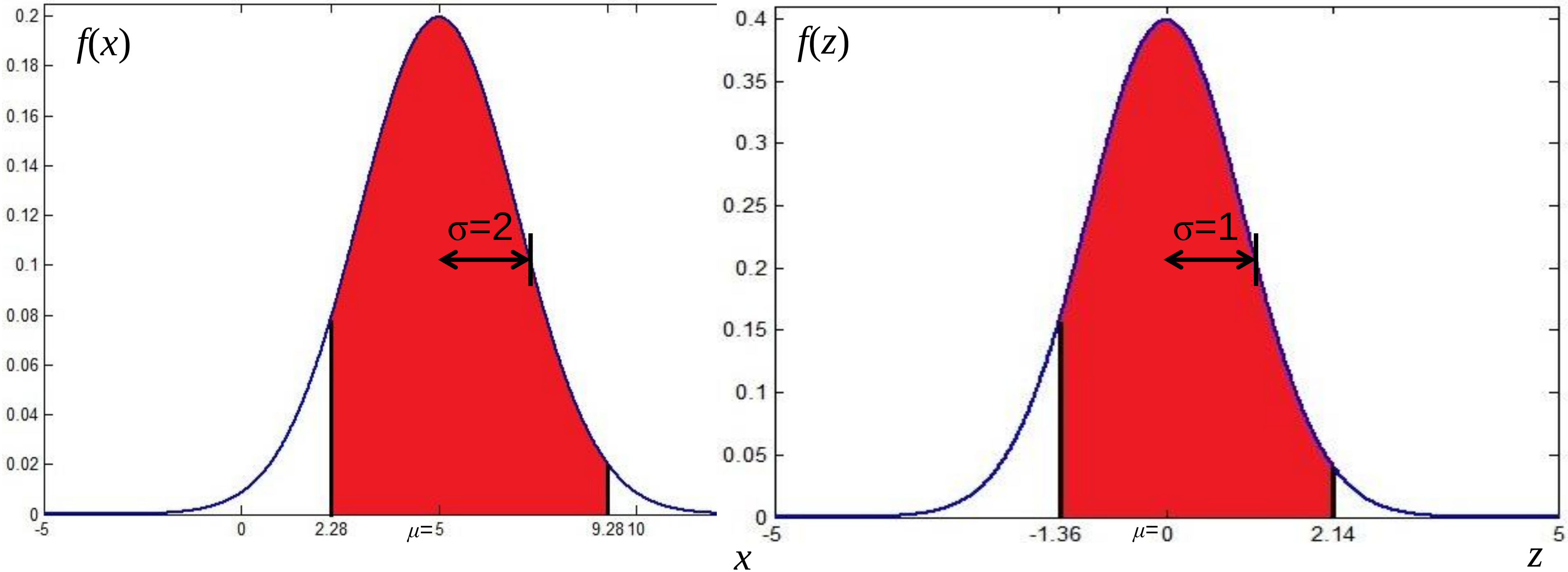
↑ ↑ ↑   ↑ ↑ ↑

2.28 5 2   9.28 5 2

# 6: Normal Probability Distributions

## 6.2 The Standard Normal Probability Distributions

**Example:** Here is a normal distribution with $\mu = 5$ and $\sigma^2 = 4$.



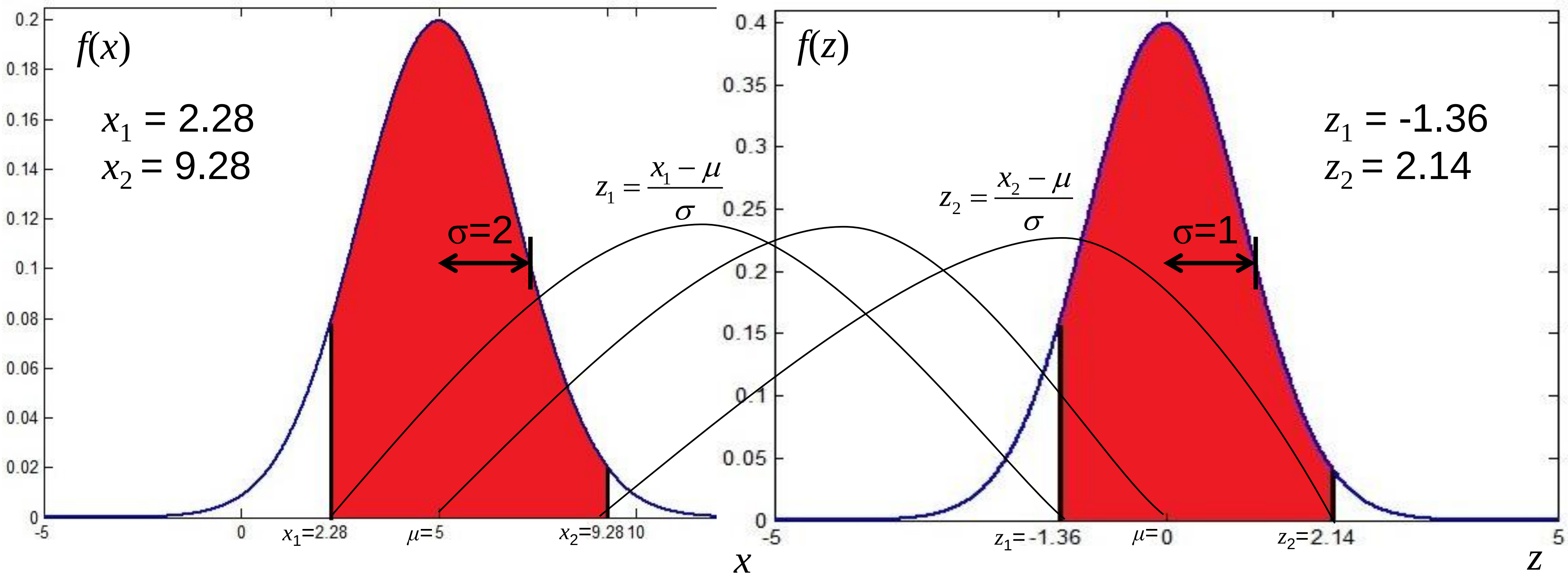

We find $z_1 = -1.36$ and $z_2 = 2.14$? Do we agree with my $z$'s?

# 6: Normal Probability Distributions

## 6.2 The Standard Normal Probability Distributions

$$z = \frac{x - \mu}{\sigma}$$

**Example:** Here is a normal distribution with $\mu = 5$ and $\sigma^2 = 4$.



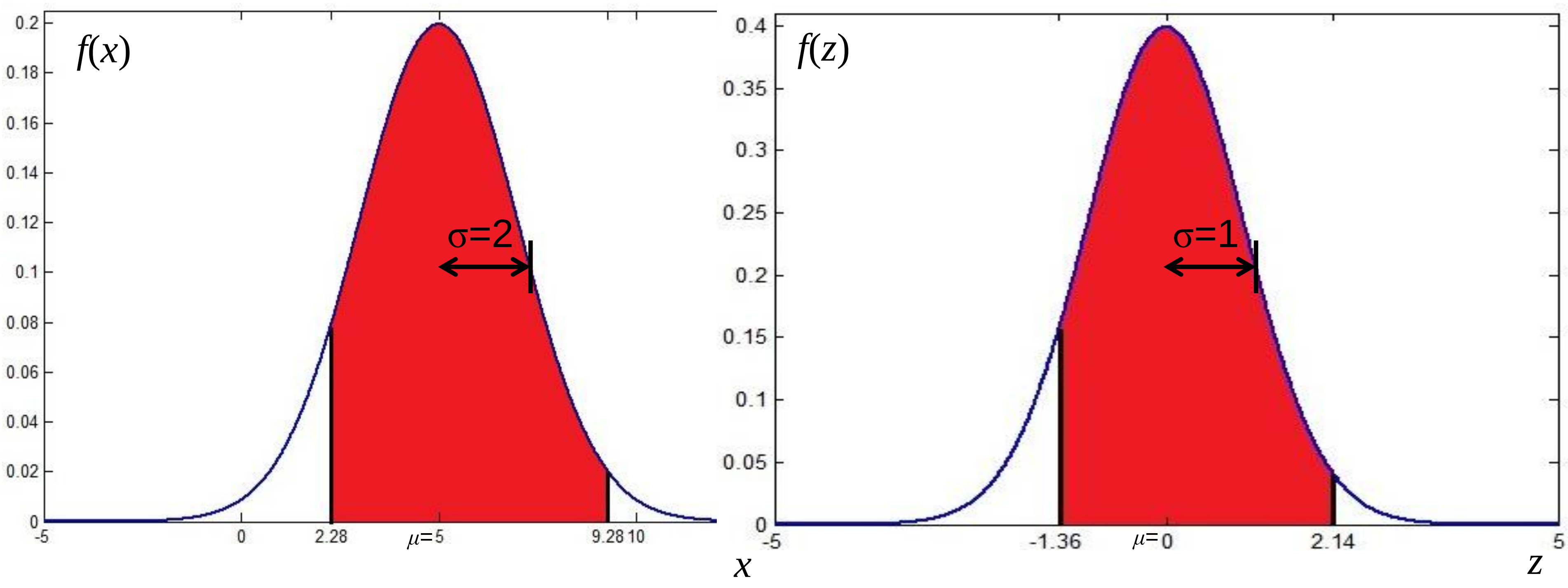

We find $z_1 = -1.36$ and $z_2 = 2.14$? Do we agree with my $z$'s?

# 6: Normal Probability Distributions

## 6.2 The Standard Normal Probability Distributions

$z_1 = -1.36$
$z_2 = 2.14$

**Example:** Here is a normal distribution with $\mu = 5$ and $\sigma^2 = 4$.




Area between $x_1$ and $x_2$ is same as the area between $z_1$ and $z_2$.

# 6: Normal Probability Distributions

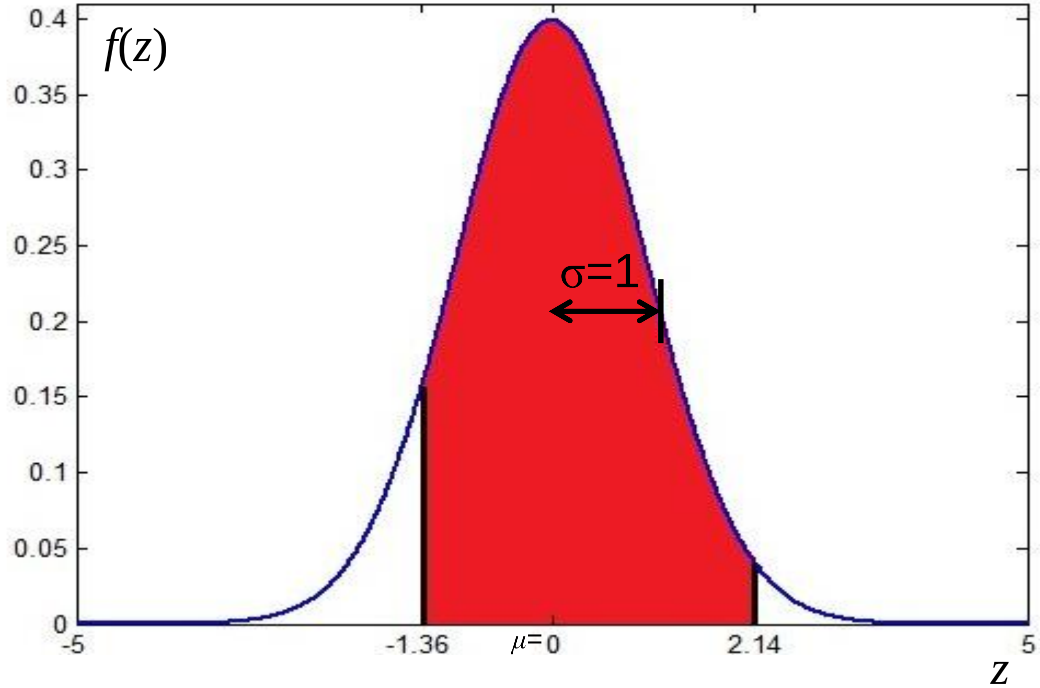
## 6.2 The Standard Normal Probability Distributions

$z_1 = -1.36$
$z_2 = 2.14$

Standard normal curve $\mu = 0$ and $\sigma^2 = 1$.

Now we can simply look up the $z$ areas in a table.

Appendix B Table 3 Page 716.



# 6: Normal Probability Distributions

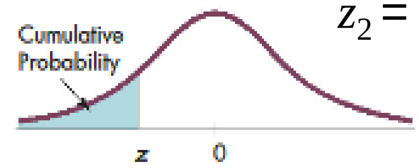
Appendix B
Table 3
Page 716

$z_1 = -1.36$
$z_2 = 2.14$

**TABLE 3**

Cumulative Areas of the Standard Normal Distribution

The entries in this table are the cumulative probabilities for the standard normal distribution $z$ (that is, the normal distribution with mean 0 and standard deviation 1). The shaded area under the curve of the standard normal distribution represents the cumulative probability to the left of a $z$-value in the **left-hand tail.**




Second Decimal Place in $z$

First Decimal Place in $z$

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −5.0 | 0.0000003 | | | | | | | | | |
| −4.5 | 0.000003 | | | | | | | | | |
| −4.0 | 0.00003 | 0.00003 | 0.00003 | 0.00003 | 0.00003 | 0.00003 | 0.00002 | 0.00002 | 0.00002 | 0.00002 |
| −3.9 | 0.00005 | 0.00005 | 0.00004 | 0.00004 | 0.00004 | 0.00004 | 0.00004 | 0.00004 | 0.00003 | 0.00003 |
| −3.8 | 0.00007 | 0.00007 | 0.00007 | 0.00006 | 0.00006 | 0.00006 | 0.00006 | 0.00005 | 0.00005 | 0.00005 |
| −3.7 | 0.00011 | 0.00010 | 0.00010 | 0.00010 | 0.00009 | 0.00009 | 0.00008 | 0.00008 | 0.00008 | 0.00008 |
| −3.6 | 0.0002 | 0.0002 | 0.0002 | 0.00014 | 0.00014 | 0.00013 | 0.00013 | 0.00012 | 0.00012 | 0.00011 |
| −3.5 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 |
| −3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| −3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 |
| −3.2 | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| −3.1 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| −3.0 | 0.0014 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| −2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| −2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| −2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| −2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0042 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| −2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| −2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0076 | 0.0073 | 0.0071 | 0.0070 | 0.0068 | 0.0066 | 0.0064 |
| −2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| −2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0126 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| −2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| −2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| −1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| −1.8 | 0.0359 | 0.0352 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| −1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| −1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| −1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |

# 6: Normal Probability Distributions

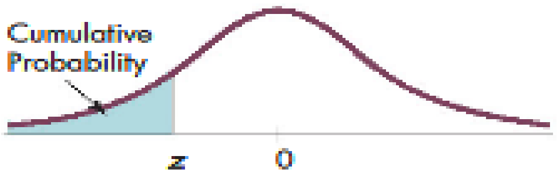
$z_1$ = -1.36
$z_2$ = 2.14

Appendix B, Table 3, Page 716

**TABLE 3**

Cumulative Areas of the Standard Normal Distribution

The entries in this table are the cumulative probabilities for the standard normal distribution $z$ (that is, the normal distribution with mean 0 and standard deviation 1). The shaded area under the curve of the standard normal distribution represents the cumulative probability to the left of a $z$-value in the **left-hand tail**.




This table gives us the area less than a $z$ value.

$P(z<z_1)$=Area less than $z_1$.

We get this from Table 3.

# 6: Normal Probability Distributions

## Appendix B, Table 3, Page 716

$z_1$ = -1.36
$z_2$ = 2.14

**TABLE 3**

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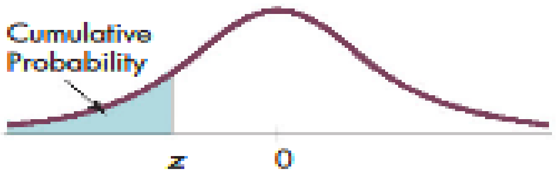
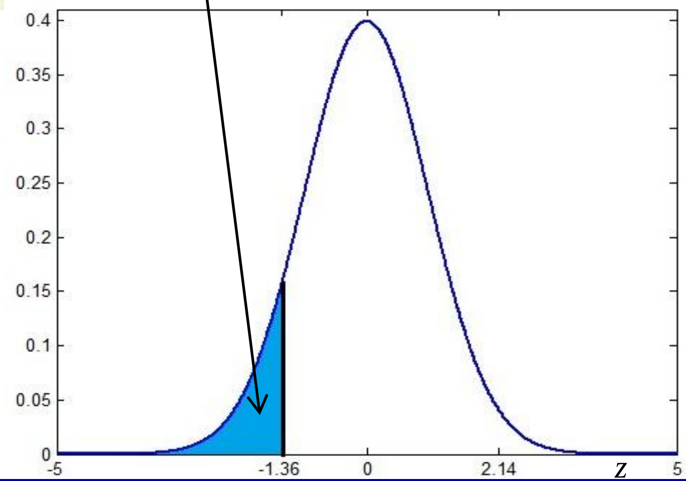



| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| −1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| −1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1094 | 0.1075 | 0.1057 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| −1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| −1.0 | 0.1587 | 0.1563 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |

$P(z<-1.36)$=Area less than -1.36.

We get this from Table 3.
Row labeled -1.3 over to column Labeled .06.



# 6: Normal Probability Distributions

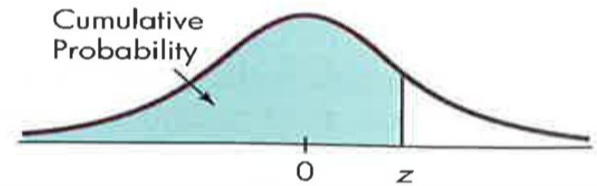
Appendix B, Table 3, Page 717

$z_1 = -1.36$
$z_2 = 2.14$

**TABLE 3**

Cumulative Areas of the Standard Normal Distribution (continued)

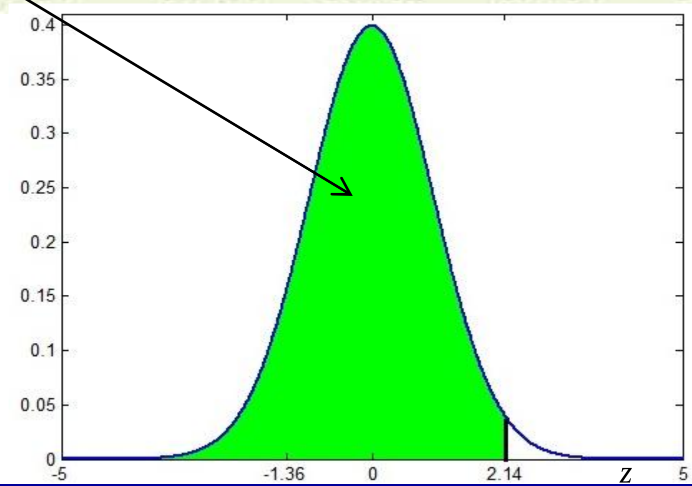
The entries in this table are the cumulative probabilities for the standard normal distribution $z$ (that is, the normal distribution with mean 0 and standard deviation 1). The shaded area under the curve of the standard normal distribution represents the cumulative probability to the left of a $z$-value in the **left-hand tail**.




| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2.0 | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |

$P(z<2.14)$=Area less than 2.14.

We get this from Table 3.
Row labeled 2.1 over to column
Labeled .04.



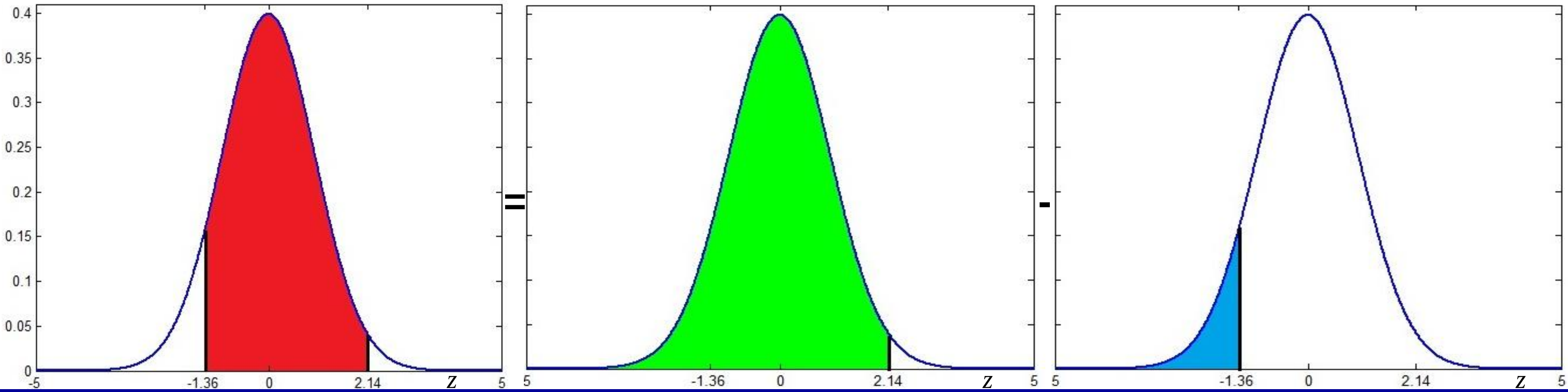
# 6: Normal Probability Distributions

Appendix B, Table 3, Page 716-717

$z_1 = -1.36$
$z_2 = 2.14$

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| −1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| −1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1094 | 0.1075 | 0.1057 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| −1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| −1.0 | 0.1587 | 0.1563 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| 2.0 | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |

$P(-1.36<z<2.14) \quad = \quad P(z<2.14) \quad - \quad P(z<-1.36)$



# 6: Normal Probability Distributions

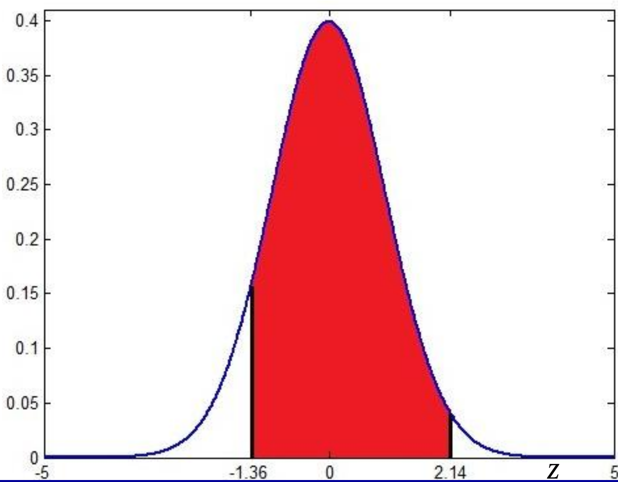
Appendix B, Table 3, Page 716-717

$z_1$ = -1.36
$z_2$ = 2.14

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| −1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| −1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1094 | 0.1075 | 0.1057 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| −1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| −1.0 | 0.1587 | 0.1563 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| 2.0 | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |

$P(-1.36<z<2.14)$ = $P(z<2.14)$ - $P(z<-1.36)$

0.8969 = 0.9838 - 0.0869

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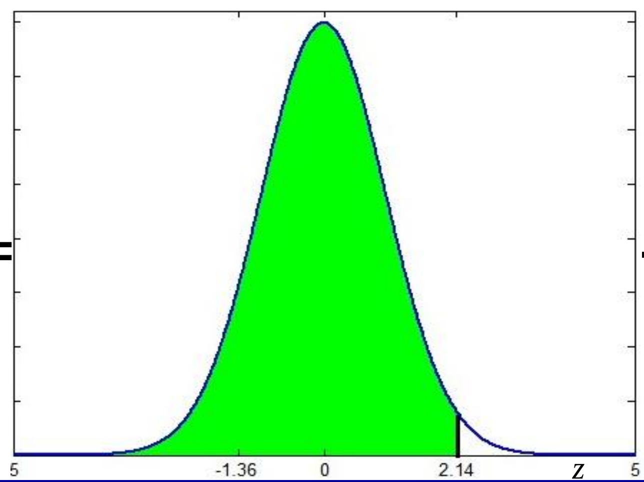


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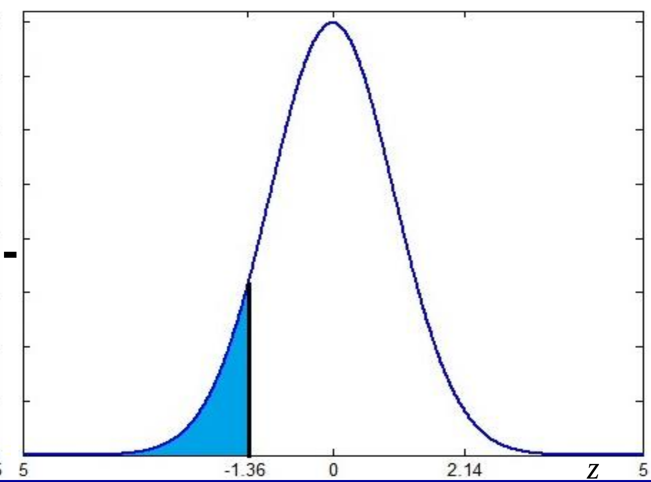


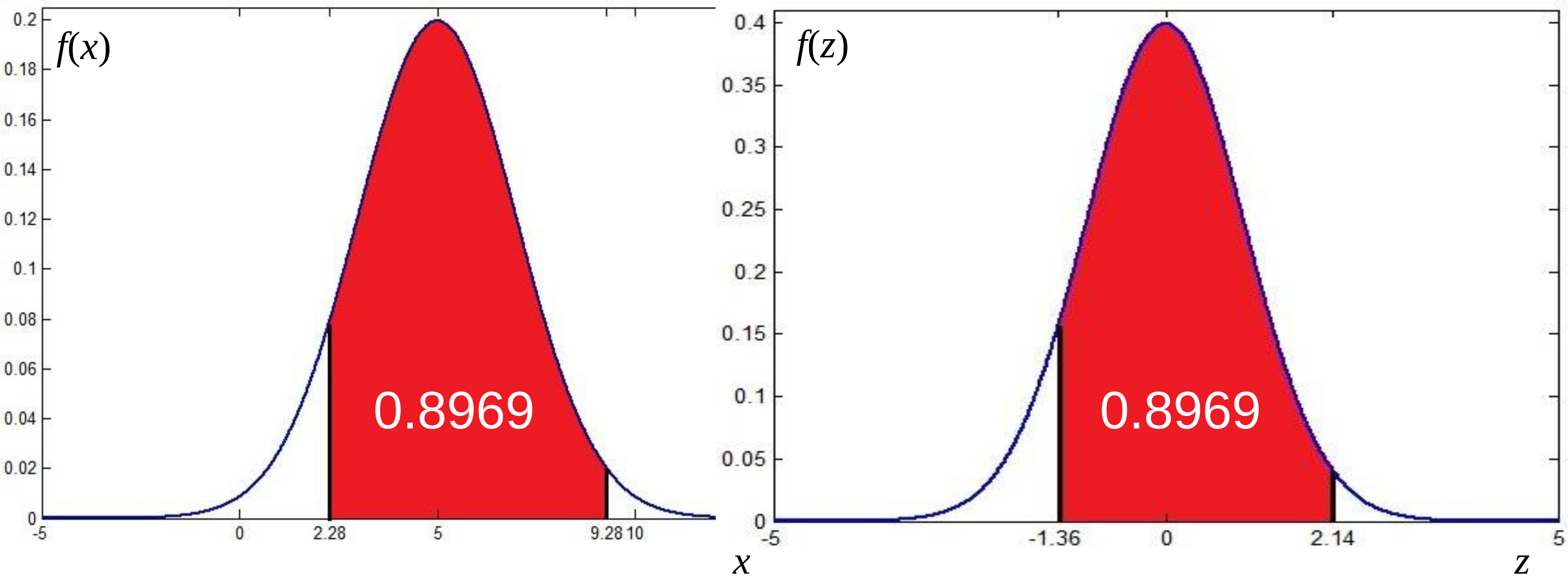

Red Area=0.8969

# 6: Normal Probability Distributions

## 6.2 The Standard Normal Probability Distributions

$z_1 = -1.36$
$z_2 = 2.14$

**Example:** Here is a normal distribution with $\mu = 5$ and $\sigma^2 = 4$.




Area between $x_1$ and $x_2$ is same as the area between $z_1$ and $z_2$.

# 6: Normal Probability Distributions

## 6.3 Applications of Normal Distributions

You may recall that in Chapter 2 we discussed a Standard Score, or $z$-score.

It was discussed then using $\bar{x}$ and $s$.
Now, we will be using $\mu$ and $\sigma$.

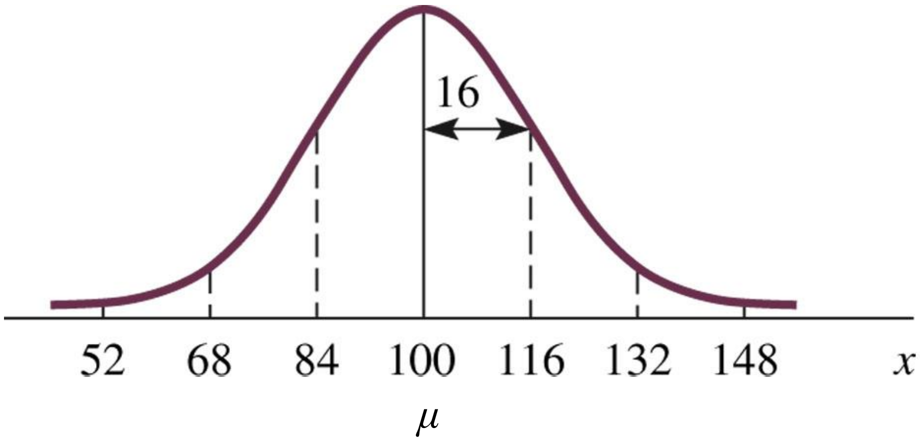
**Standard score, or z-score:** The position a particular value of $x$ has relative to the mean, measured in standard deviations.

$$z_i = \frac{x \text{ - mean of } x}{\text{std. dev. of } x} = \frac{x_i - \mu}{\sigma} \qquad (6.3)$$

# 6: Normal Probability Distributions

## 6.3 Applications of Normal Distributions

Assume that IQ scores $x$ are normally distributed with a mean $\mu$ of 100 and a standard deviation $\sigma$ of 16.




Figures from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions

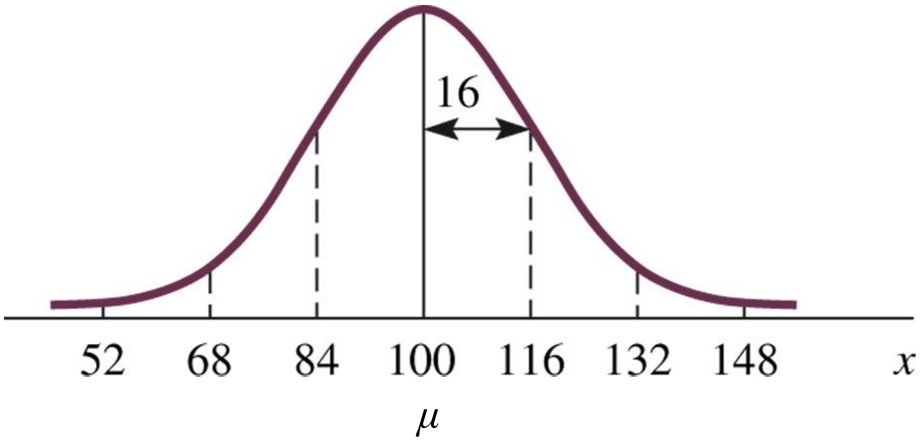
## 6.3 Applications of Normal Distributions

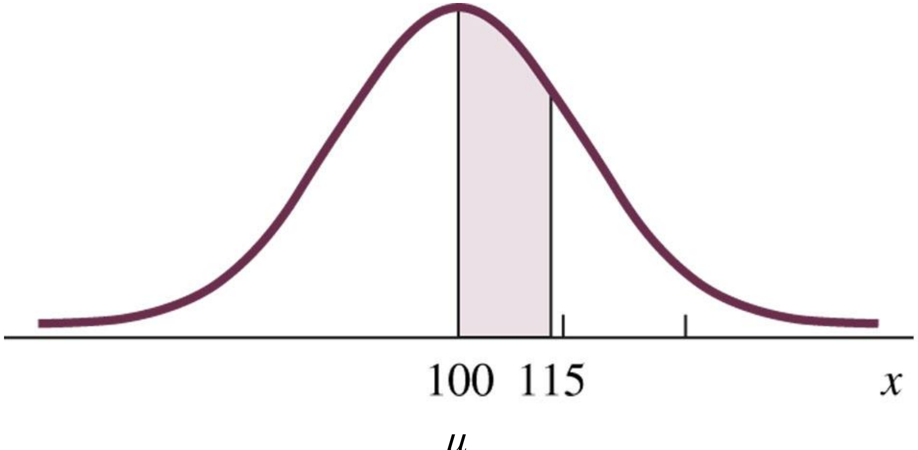
**Example:**

Assume that IQ scores $x$ are normally distributed with a mean $\mu$ of 100 and a standard deviation $\sigma$ of 16.

If a person is picked at random, what is the probability that his or her IQ is between 100 and 115?

i.e. $P(100 < x < 115)$ ?




Figures from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions

## 6.3 Applications of Normal Distributions

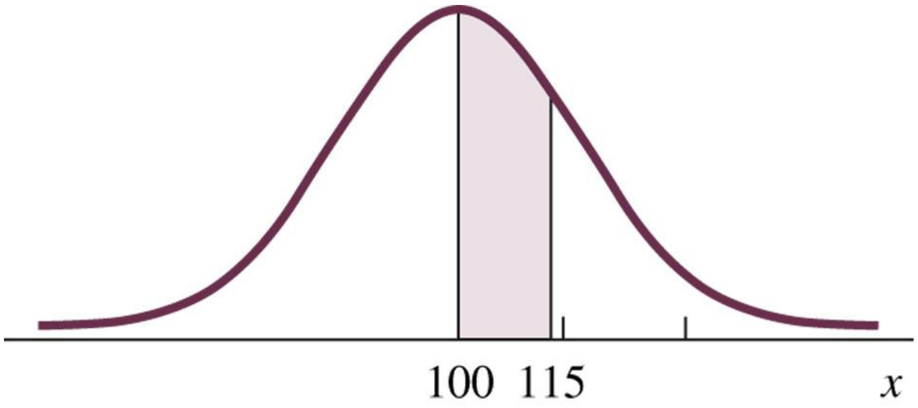
IQ scores normally distributed $\mu$=100 and $\sigma$=16.

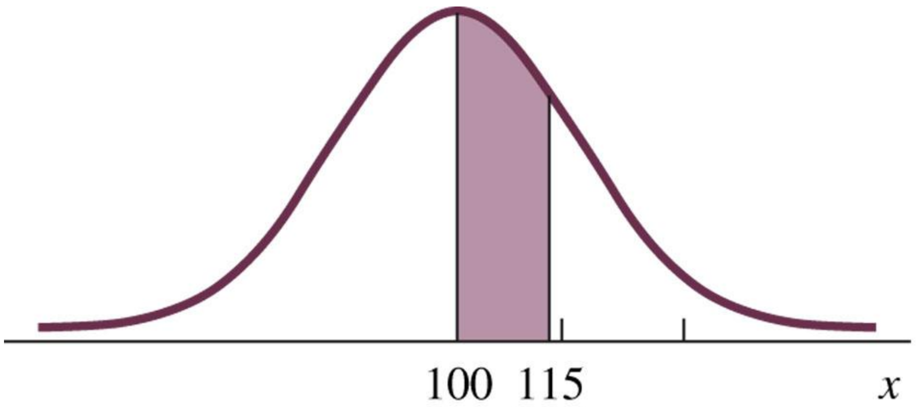
$$P(100 < x < 115) = P(z_1 < z < z_2)$$

$$z = \frac{x-\mu}{\sigma} \qquad x_1 = 100 \quad x_2 = 115$$

$$z_1 = \frac{x_1-\mu}{\sigma} =$$

$$z_2 = \frac{x_2-\mu}{\sigma} =$$




Figures from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions

## 6.3 Applications of Normal Distributions

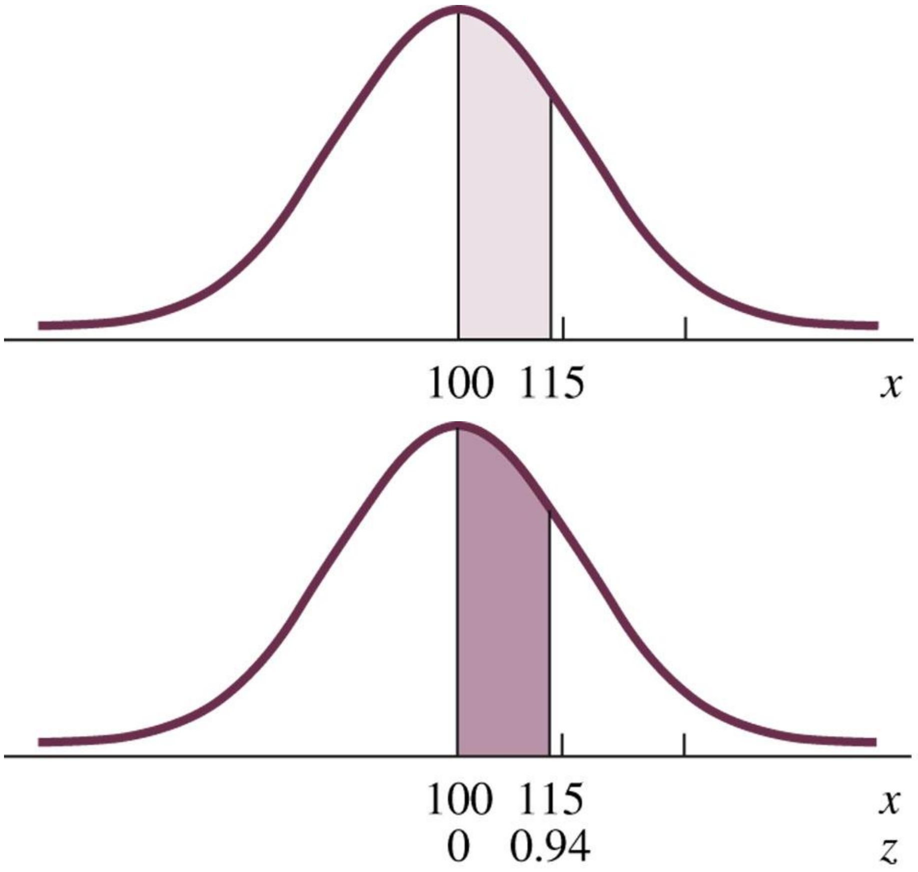
IQ scores normally distributed $\mu$=100 and $\sigma$=16.

$$P(100 < x < 115)$$

$$z = \frac{x-\mu}{\sigma} \qquad x_1 = 100 \quad x_2 = 115$$

$$z_1 = \frac{x_1-\mu}{\sigma} = \frac{100-100}{16} = 0$$

$$z_2 = \frac{x_2-\mu}{\sigma} = \frac{115-100}{16} = 0.94$$




Figures from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions
## 6.3 Applications of Normal Distributions

IQ scores normally distributed
$\mu$=100 and $\sigma$=16.

$$P(100 < x < 115) = P(0 < z < 0.94)$$

Now we can use the table.

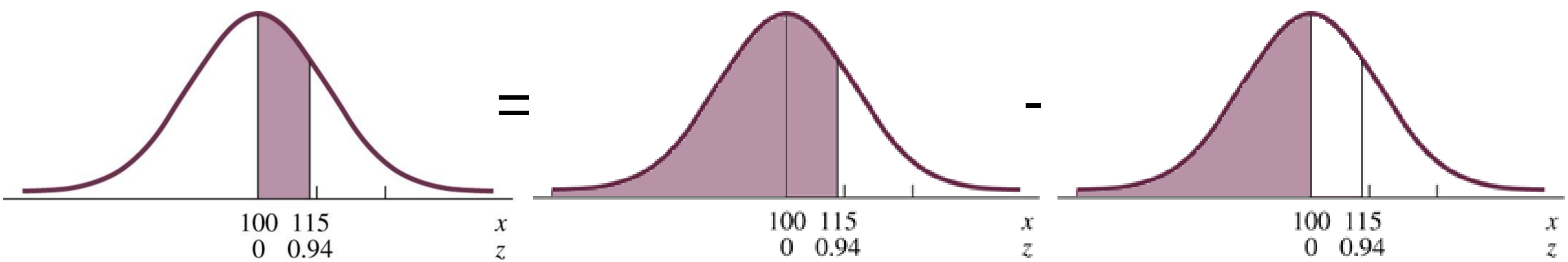
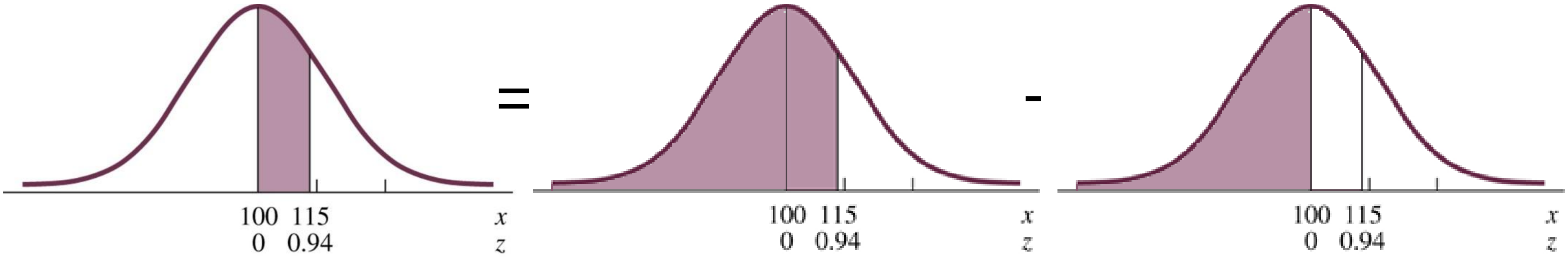



Figure from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions

## 6.3 Applications of Normal Distributions

Now we can use the table.

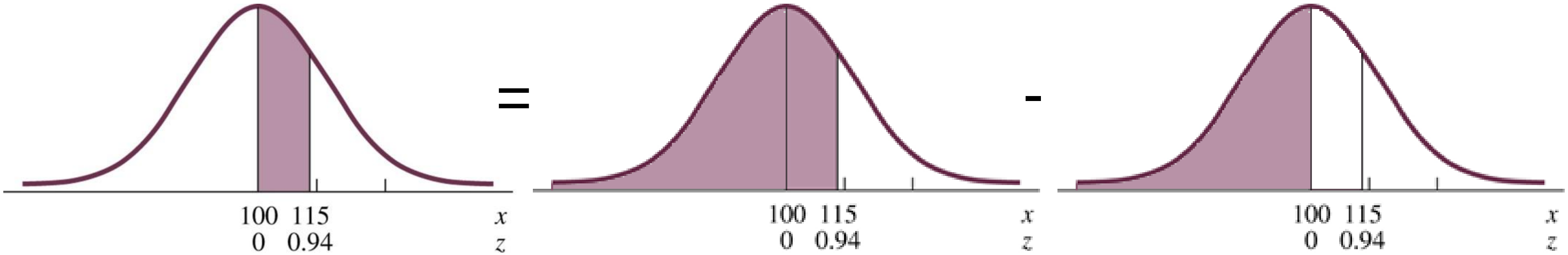



$$P(0 < z < 0.94) =$$

Figures from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions
## 6.3 Applications of Normal Distributions

Now we can use the table.




$$P(0 < z < 0.94) = P(z < 0.94) - P(z < 0)$$
$$= 0.8264 - .5$$
$$= 0.3264$$

Figures from Johnson & Kuby, 2012.

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |

# 6: Normal Probability Distributions
## 6.4 Notation

We can use the table in reverse.

Before we had a $z$ value then looked up the probability (area) less than $z$.

Now we will have a probability (area), call it $\alpha$, and want to know the $z$ value, call it $z(\alpha)$, that has a probability (area) of $\alpha$ larger than it.

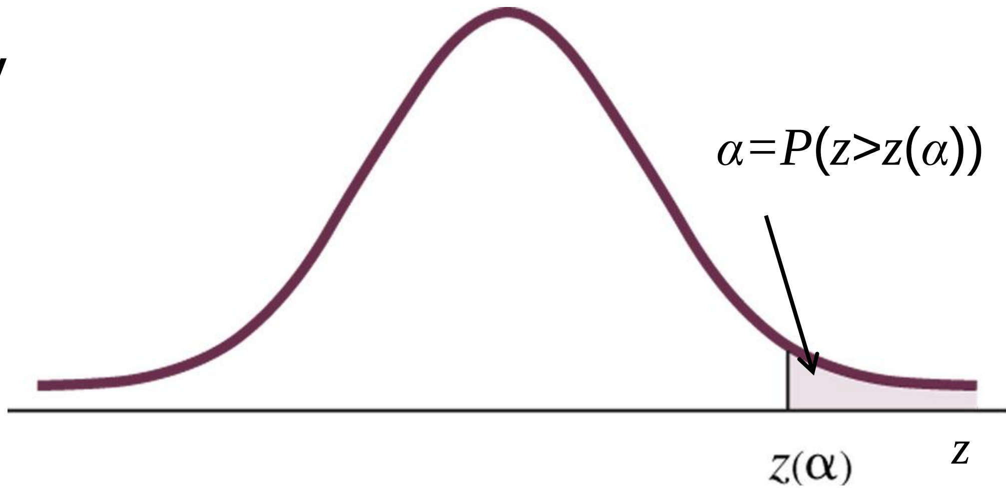



Figure from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions

## 6.4 Notation

**Example:**

Let $\alpha$=0.05. Let's find $z(0.05)$.

$P(z>z(0.05))$=0.05.

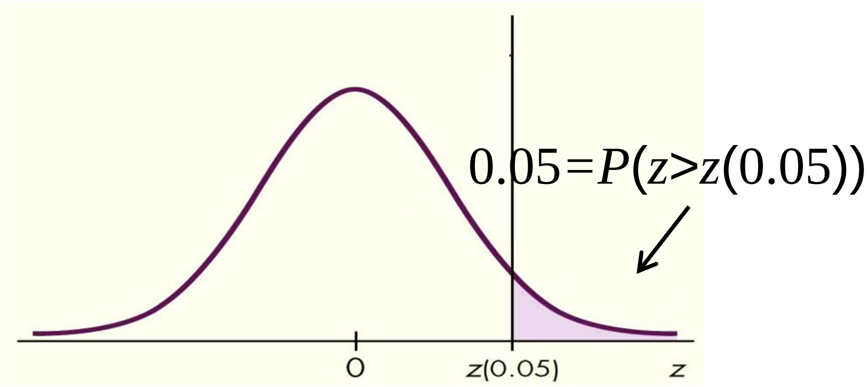



Figure from Johnson & Kuby, 2012.

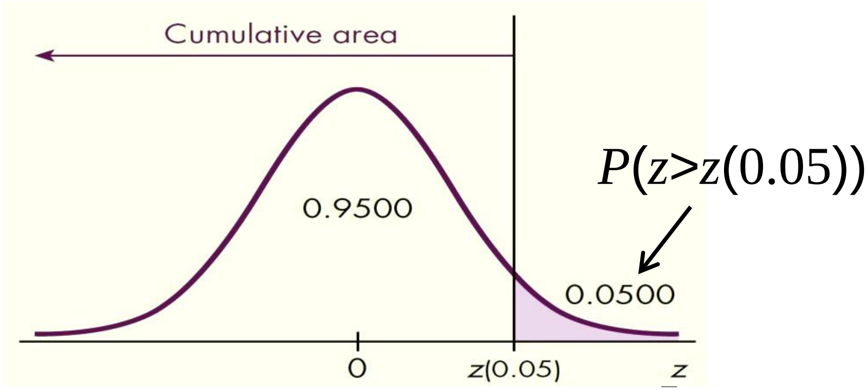
# 6: Normal Probability Distributions

## 6.4 Notation

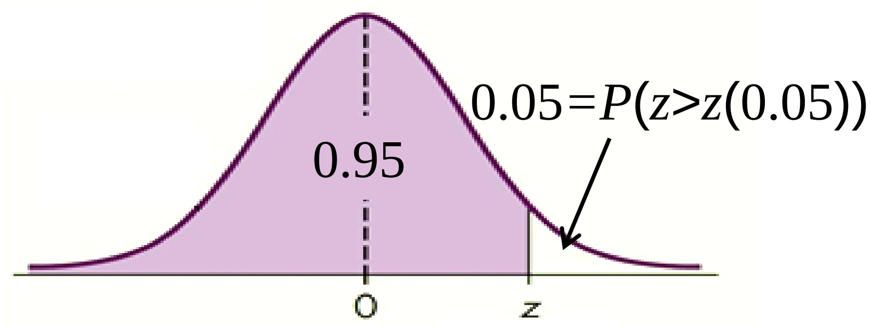
**Example:**

Let $\alpha$=0.05. Let's find $z$(0.05).

$P(z>z(0.05))$=0.05.



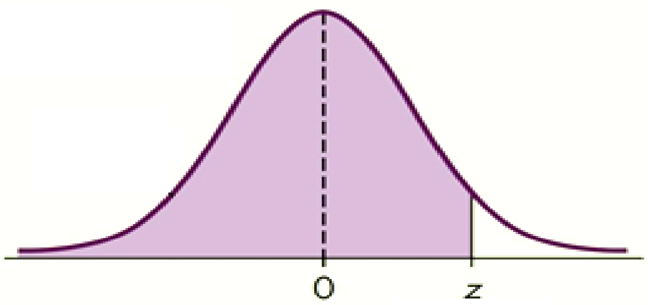

Same as finding $P(z<z(0.05))$=1-0.05.




Figures from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions

## 6.4 Notation




**Example:**

Same as finding $P(z<z(0.05))=0.95$.

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5754 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7258 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7518 | 0.7549 |
| 0.7 | 0.7580 | 0.7612 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7996 | 0.8023 | 0.8051 | 0.8079 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| ⋮ | | | | | | | | | | |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9430 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9485 | 0.949 | .9500 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9700 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9762 | 0.9767 |
| 2.0 | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |

1.645

Figures from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions

## 6.5 Normal Approximation of the Binomial Distribution

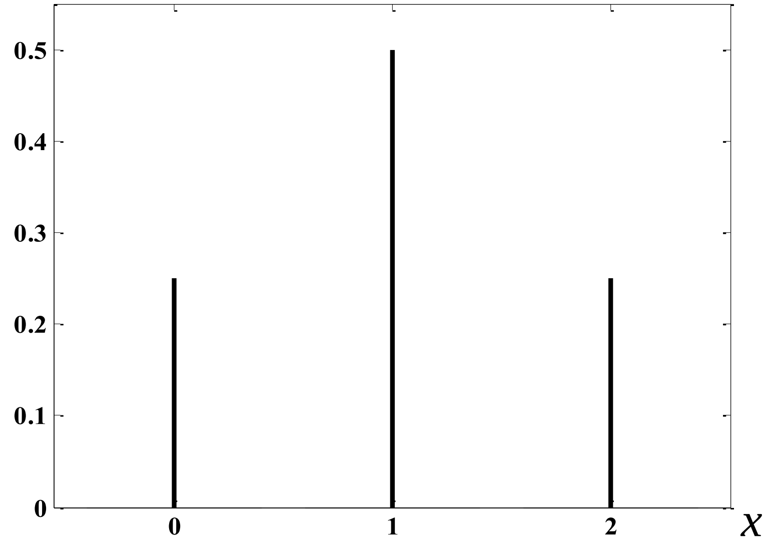
In Chapter 5 we discussed the binomial distribution

$$P(x)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x} \qquad x=0,\ldots,n$$

$x$ = # of heads when we flip a coin $n$ times

$n$=2
$p$=1/2

| $x$ | $P(x)$ |
|---|---|
| 0 | $\frac{1}{4}$ |
| 1 | $\frac{1}{2}$ |
| 2 | $\frac{1}{4}$ |



# 6: Normal Probability Distributions

## 6.5 Normal Approximation of the Binomial Distribution

If we flip the coin a large number of times

$$P(x)=\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x} \qquad x=0,\ldots,n$$

$x$ = # of heads when we flip a coin $n$ times

$n$=14
$p$=1/2

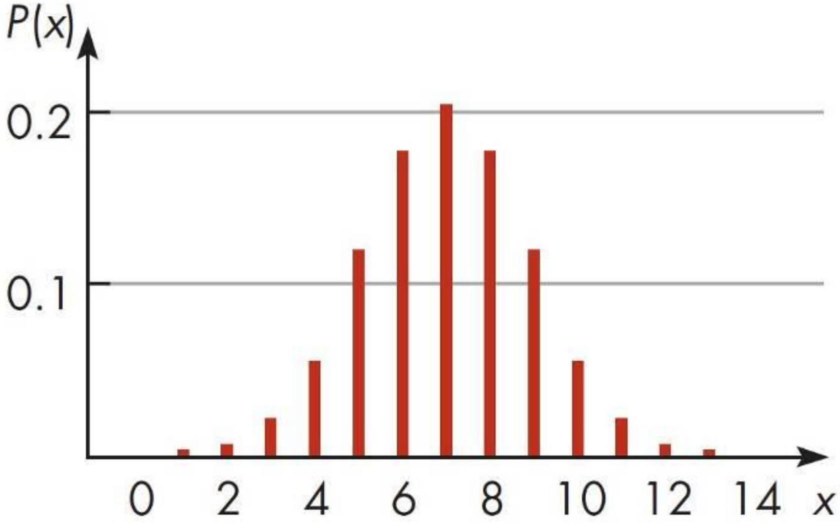



Figure from Johnson & Kuby, 2012.

It gets tedious to find the $n$=14 probabilities!

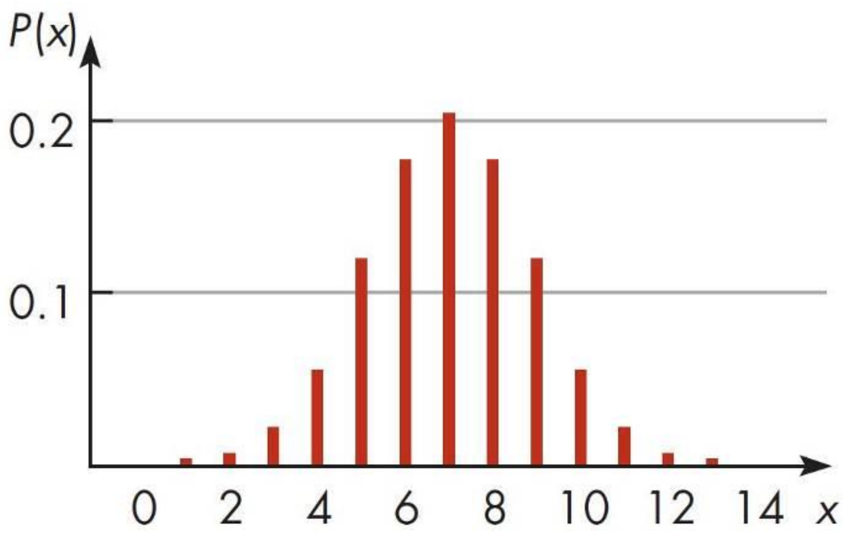
# 6: Normal Probability Distributions

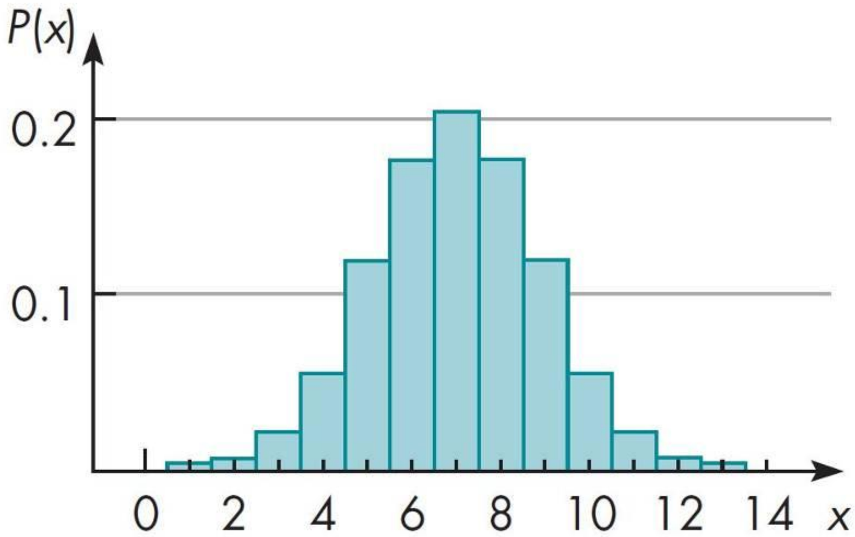
## 6.5 Normal Approximation of the Binomial Distribution

$n$=14
$p$=1/2

It gets tedious to find the $n$=14 probabilities!

So what we can do is use a histogram representation,



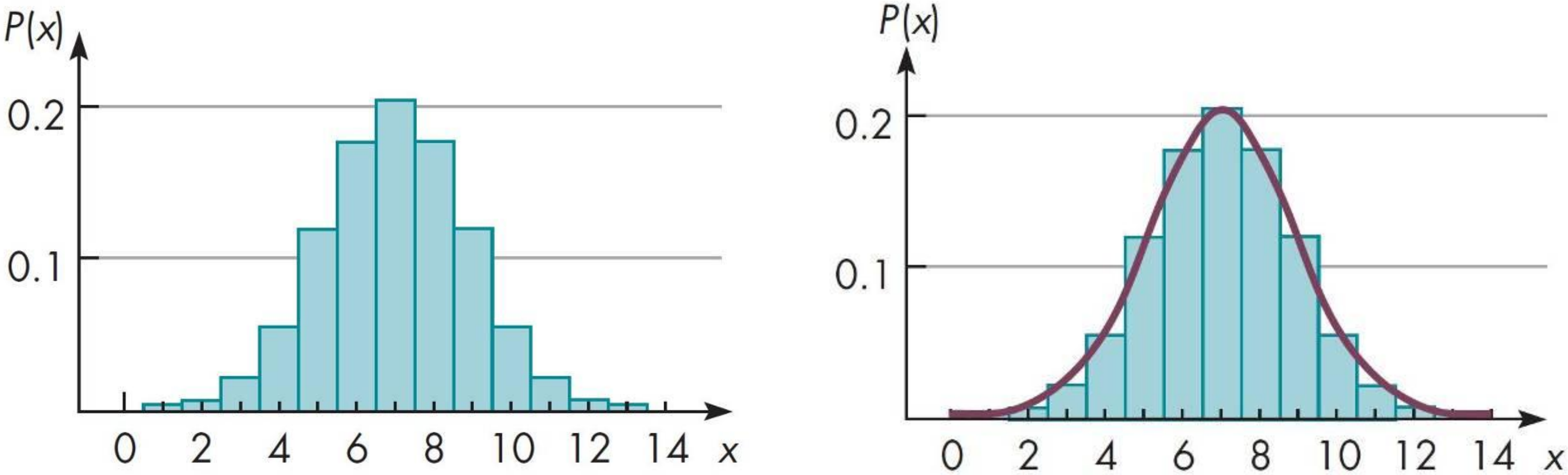

Figures from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions

## 6.5 Normal Approximation of the Binomial Distribution

So what we can do is use a histogram representation, $n$=14 $p$=1/2

Then approximate binomial probabilities with normal areas.




Figures from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions

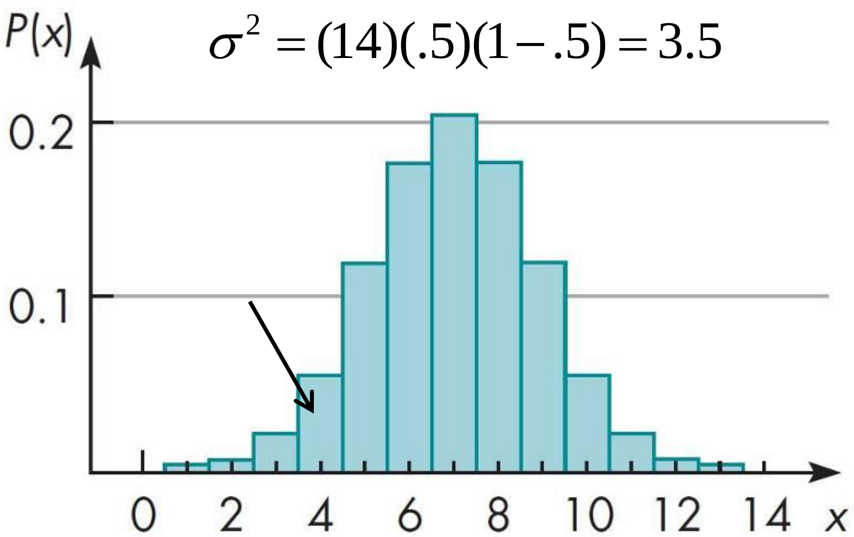
## 6.5 Normal Approximation of the Binomial Distribution

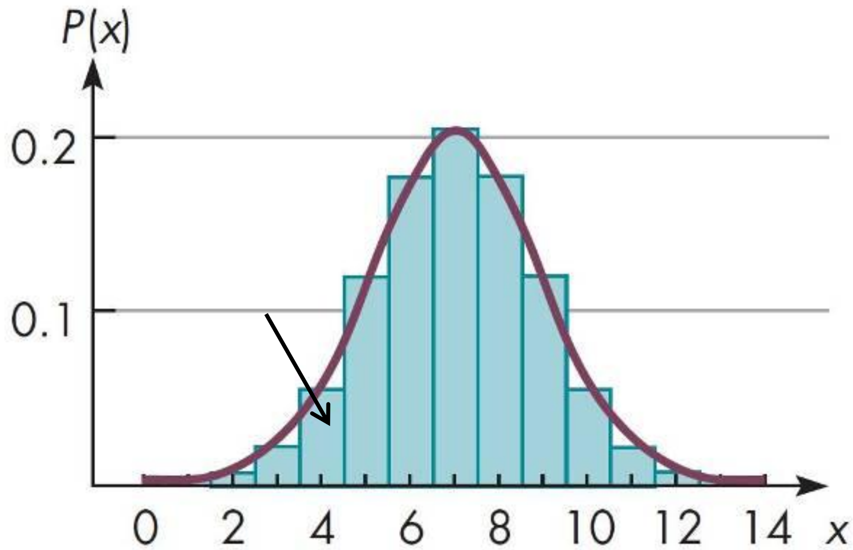
Approximate binomial probabilities with normal areas.
Use a normal with $\mu = np,\ \sigma^2 = np(1-p)$

$n$=14
$p$=1/2

$$\mu = (14)(.5) = 7$$

$$\sigma^2 = (14)(.5)(1-.5) = 3.5$$




Figures from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions

## 6.5 Normal Approximation of the Binomial Distribution

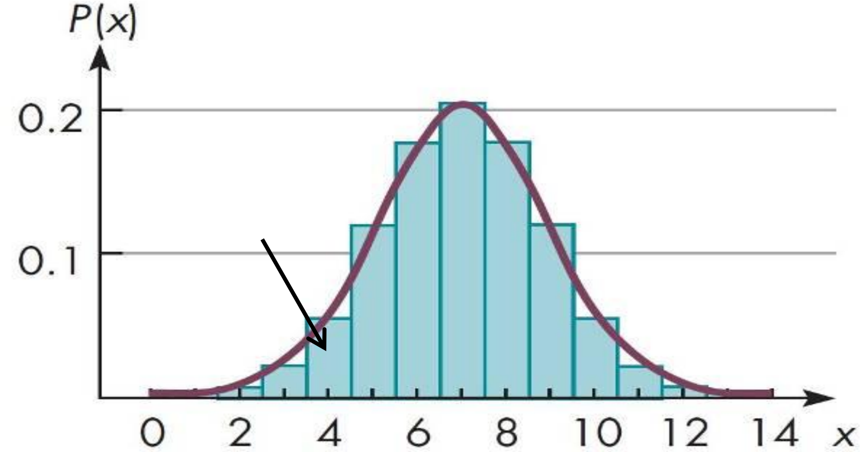
$n$=14, $p$=1/2

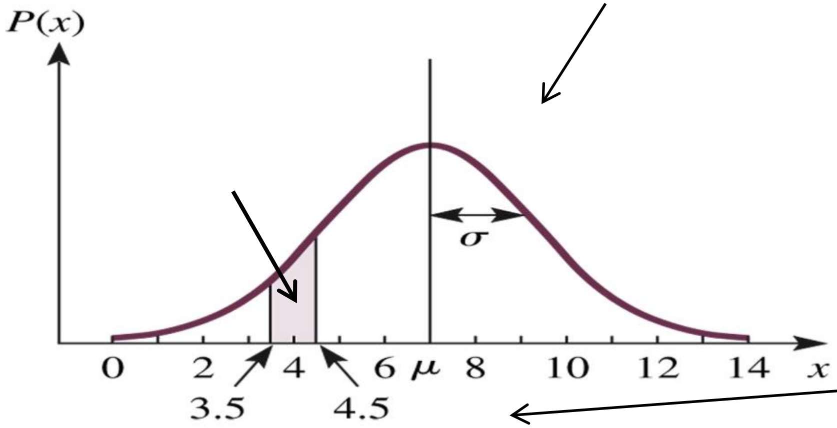
We then approximate binomial probabilities with normal areas.

$P(x=4)$ from the binomial formula

is approximately $P(3.5<x<4.5)$

from the normal with $\mu=7,\ \sigma^2=3.5$




the ±.5 is called a "continuity correction"

Figures from Johnson & Kuby, 2012.

# 6: Normal Probability Distributions

*n*=14, *p*=1/2

## 6.5 Normal Approximation of the Binomial Distribution

From the binomial formula

$$P(4)=\frac{14!}{4!(14-4)!}(.5)^{4}(1-.5)^{14-4}$$

$$P(x=4)=0.061$$

---

From the Normal Distribution

$$P(3.5<x<4.5)$$

$$\mu=7,\ \sigma^{2}=3.5$$

$$\sigma=1.87$$

$$z_1=\frac{x_1-\mu}{\sigma}=$$

$$z_2=\frac{x_2-\mu}{\sigma}=$$

# 6: Normal Probability Distributions

*n*=14, *p*=1/2

## 6.5 Normal Approximation of the Binomial Distribution

From the binomial formula

$$P(4)=\frac{14!}{4!(14-4)!}(.5)^4(1-.5)^{14-4}$$

$$P(x=4)=0.061$$

$$P(-1.87<z<1.34)=0.0594$$

From the Normal Distribution

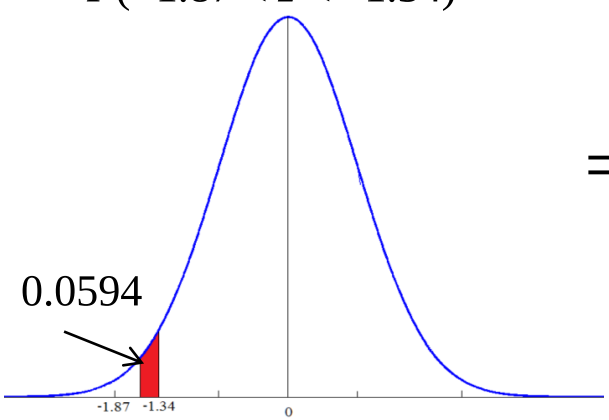
$$P(3.5<x<4.5) \qquad \mu=7,\ \sigma^2=3.5$$

$$\sigma=1.87$$

$$z_1=\frac{x_1-\mu}{\sigma}=\frac{3.5-7}{1.87}=\text{-}1.87$$

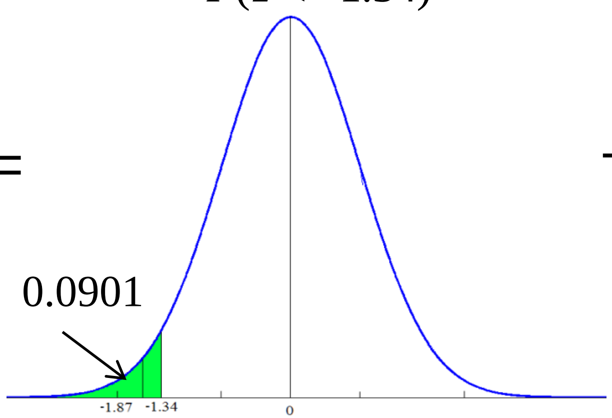
$$z_2=\frac{x_2-\mu}{\sigma}=\frac{4.5-7}{1.87}=-1.34$$

$$P(-1.87<z<-1.34)$$




=

$$P(z<-1.34)$$

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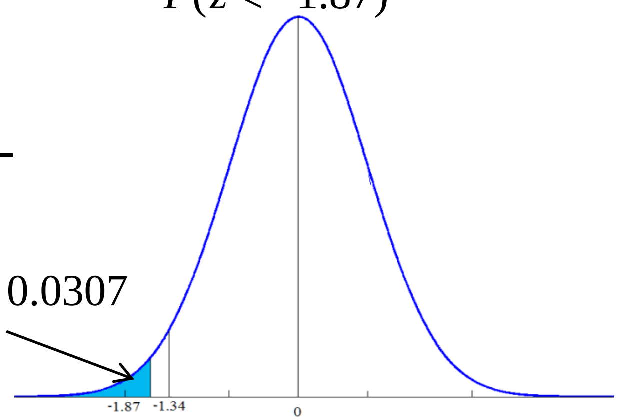
$$P(z<-1.87)$$



# 6: Normal Probability Distributions

Questions?

Homework: Read Chapter 6.1-6.2

Web Assign

Chapter 6 # 7a&b, 9a&b, 13a, 19, 29, 31, 33, 41, 45, 47, 53, 61, 75, 95, 99

Not homework, but maybe fun to watch: NETFLIX

# Return Exam 1.