Class 23: FMRI Application

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Outline

Background of FMRI

Estimating Mean and Variance

Hypothesis Testing on Mean

Hypothesis Testing on Variance

Hypothesis Testing on Difference in Means

Discussion

Background of FMRI

A younger Dr. Rowe at the MRI machine trying to look smart.

Background of FMRI

FMRI (Functional magnetic resonance imaging) is a non-invasive MRI technique to obtain volume images of the brain while a subject/patient is performing a task.

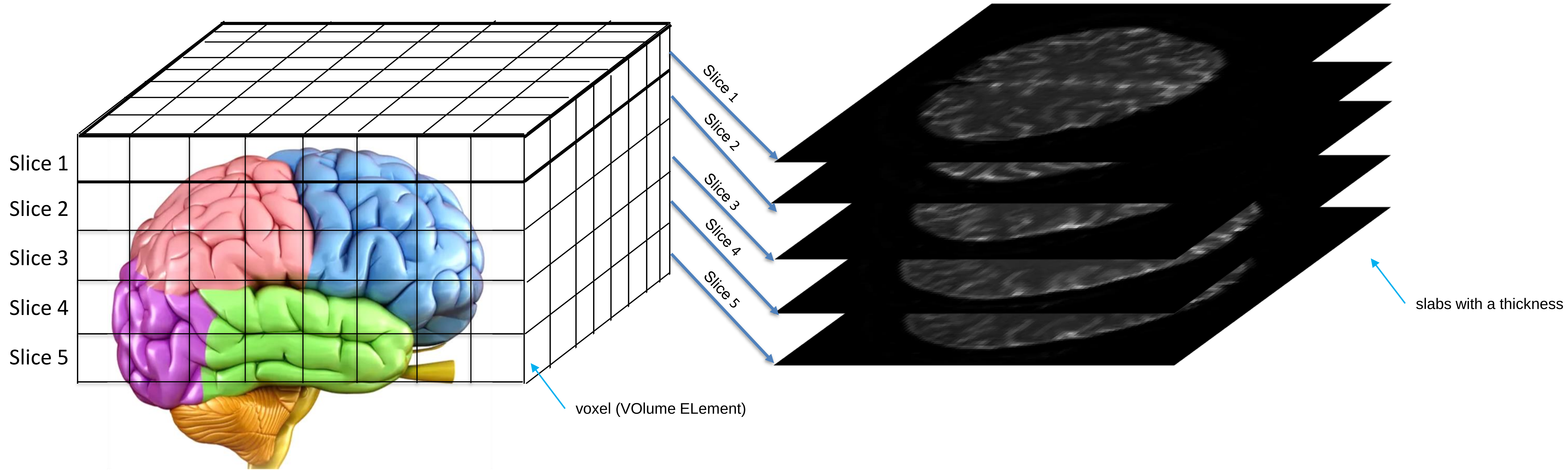
There are n volumes of the brain measured during the scan.

Each volume image parcellated into a lattice cuboids called voxels.

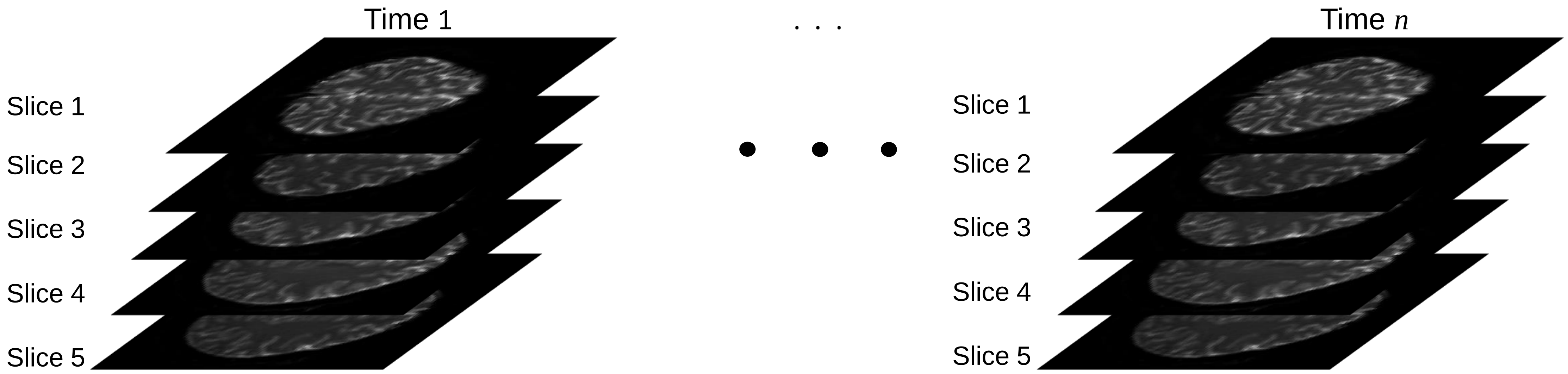
There are n time measurements in each voxel forming a time series.

Statistics are performed on the n measurements in each voxel.

Background of FMRI



Volume Time Series



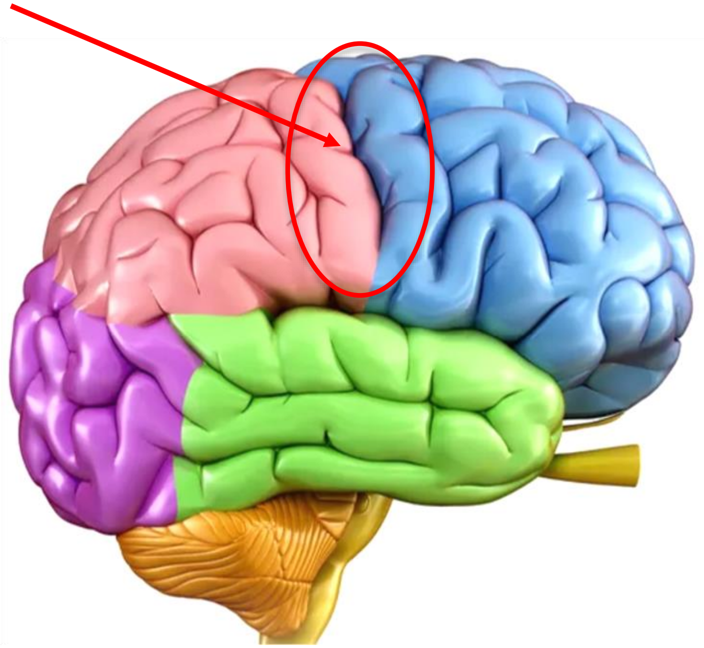
Background of FMRI

Dr. Rowe’s research is to develop technology for FMRI brain imaging.

The data shown here is from an old scan to illustrate statistical methods.

There are n brain volumes measured during bilateral finger tapping.

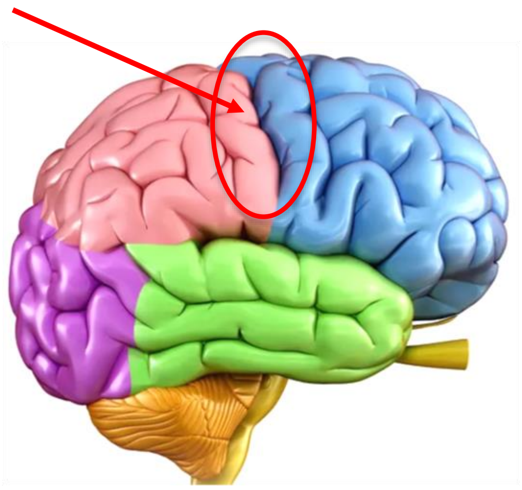
Brain activation is expected to be seen in the motor cortex.

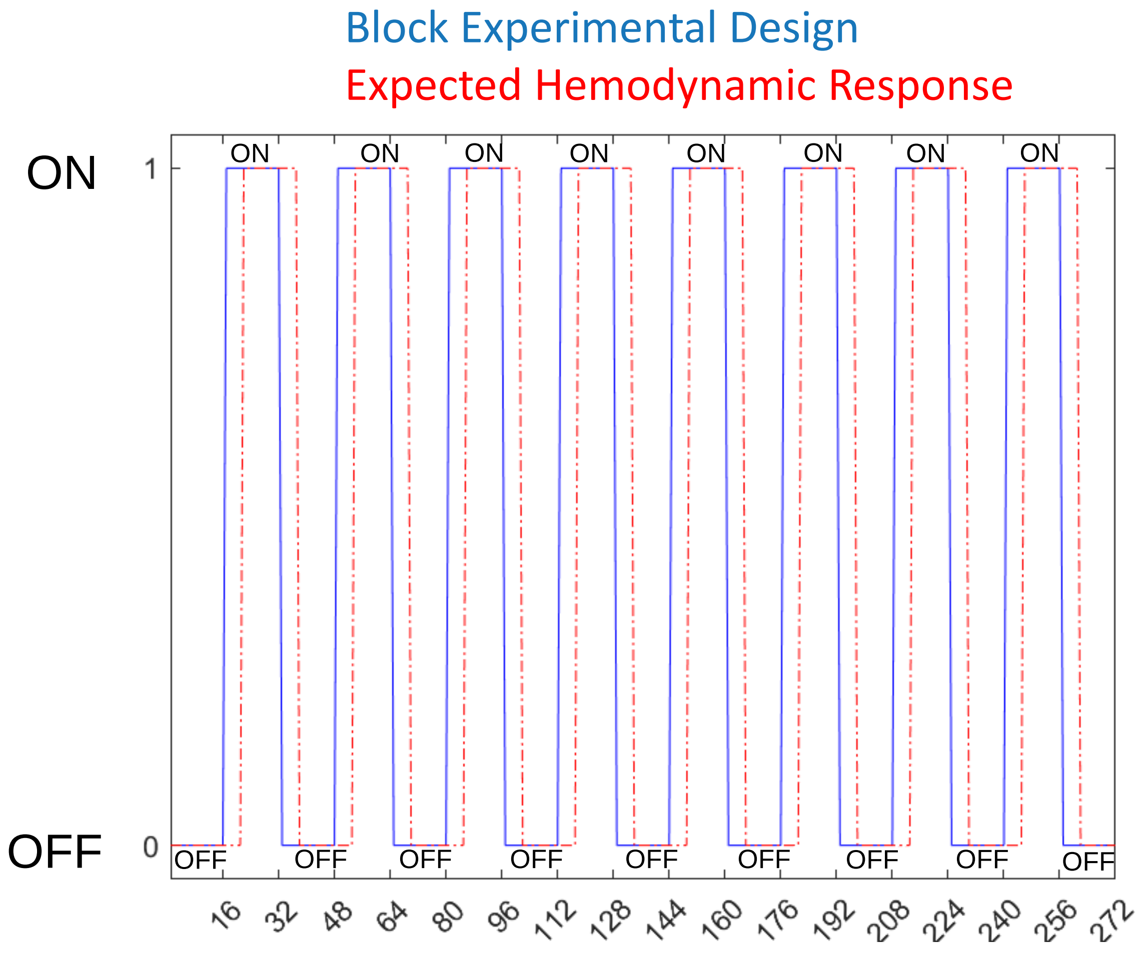


Background of FMRI

A bilateral sequential finger-tapping experiment was performed in a block design with 16-s off (no tap) followed by eight epochs of 16-s on (tap) and 16-s off (no-tap).

Tap Fingers



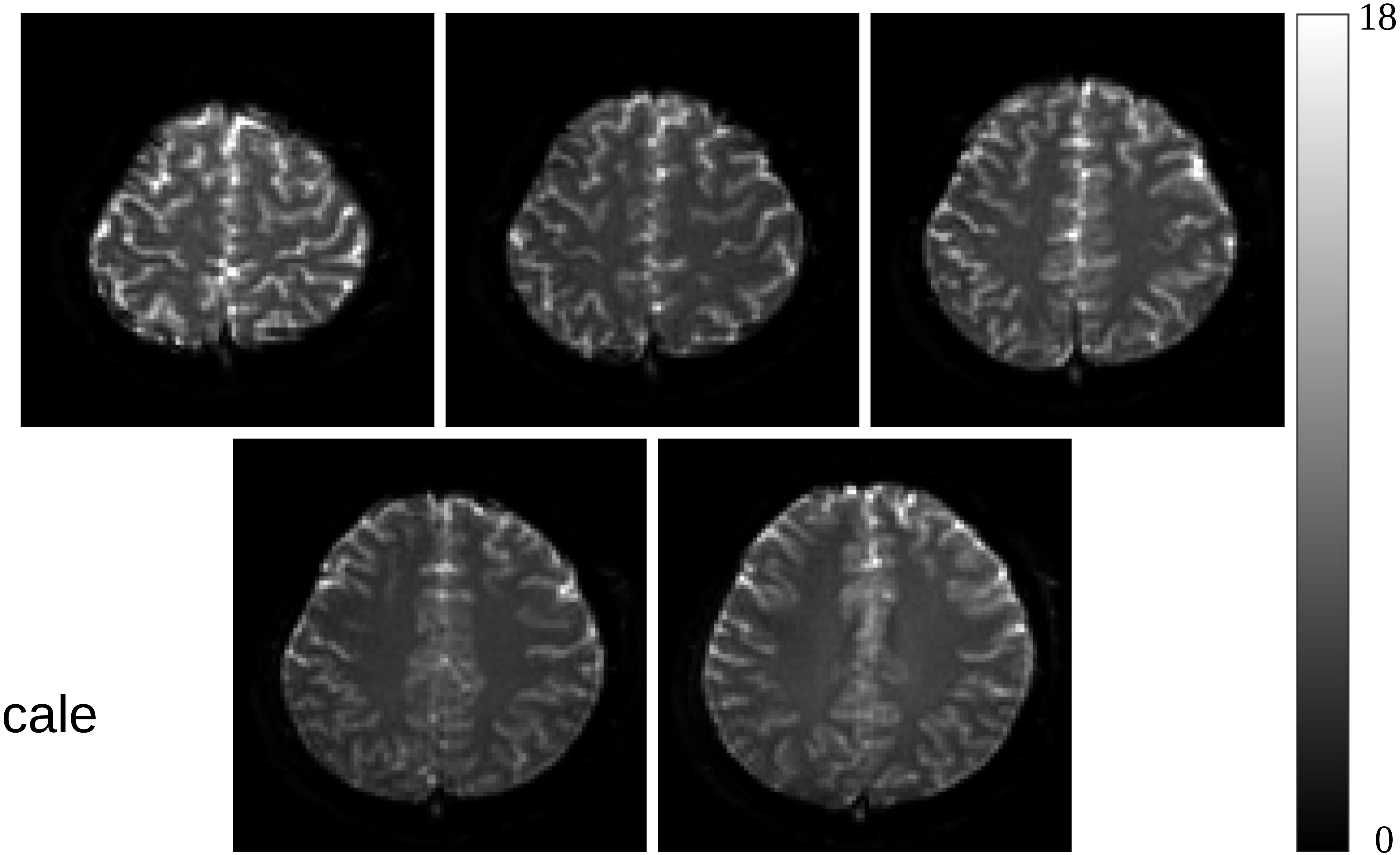


Rowe DB, Logan BR. A complex way to compute fMRI activation. Neuroimage 23(3):1078-1092 (2004).

Background of FMRI

First volume image is always brightest and I use for an anatomical underlay.

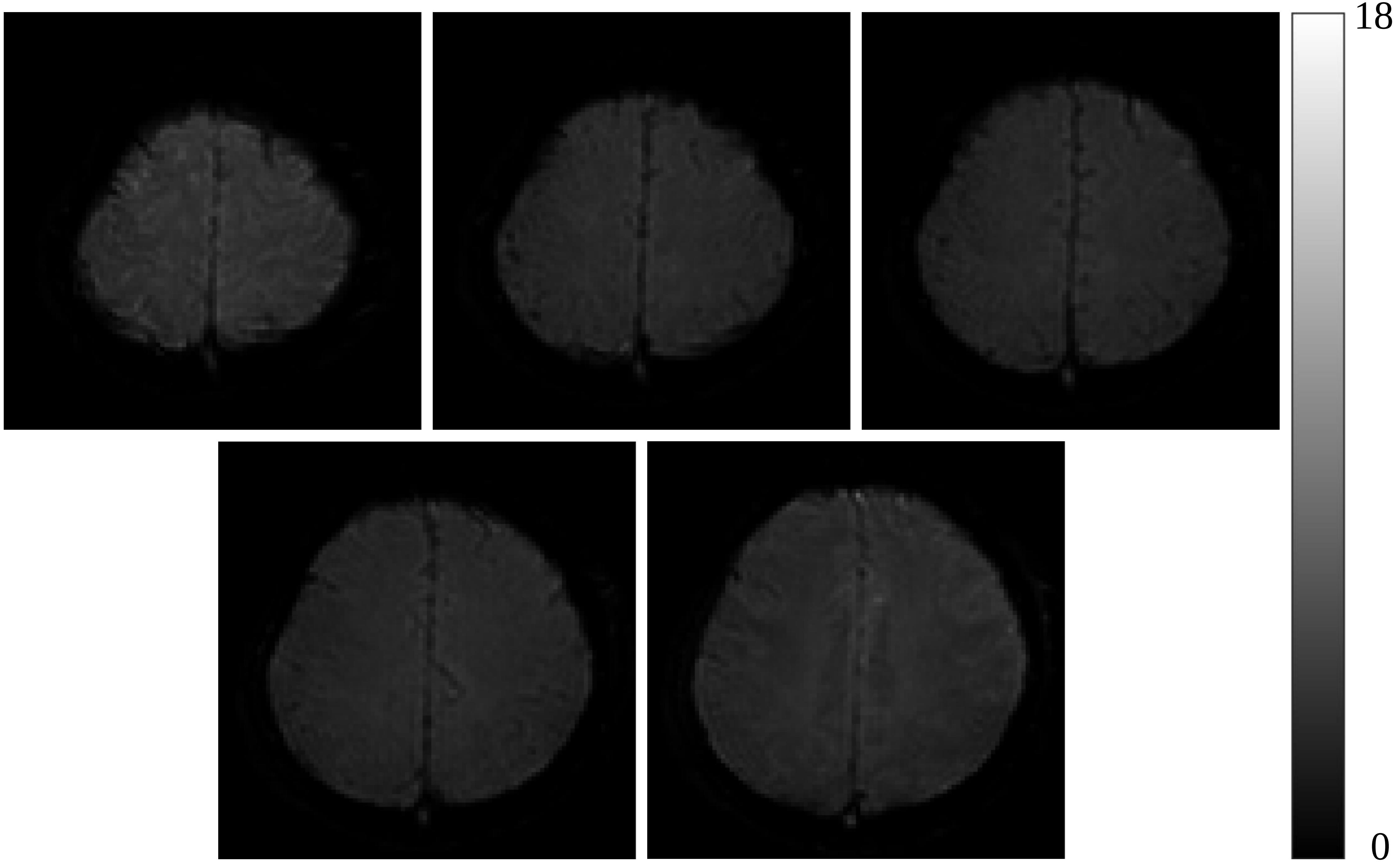
FMRI uses T_2* images so greyscale is reversed.



Background of FMRI

Pixel intensity decreases to steady state by around the fourth volume image.

So we delete the first 3.

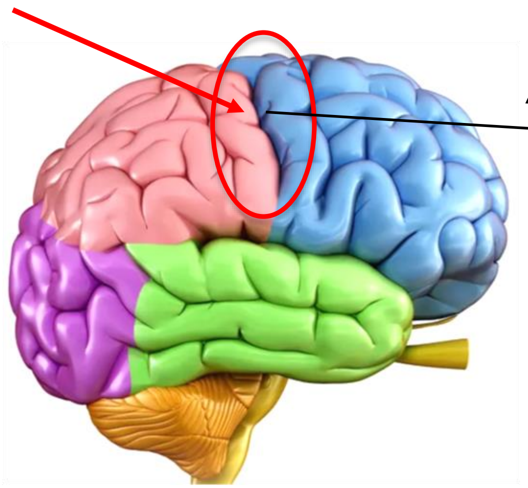


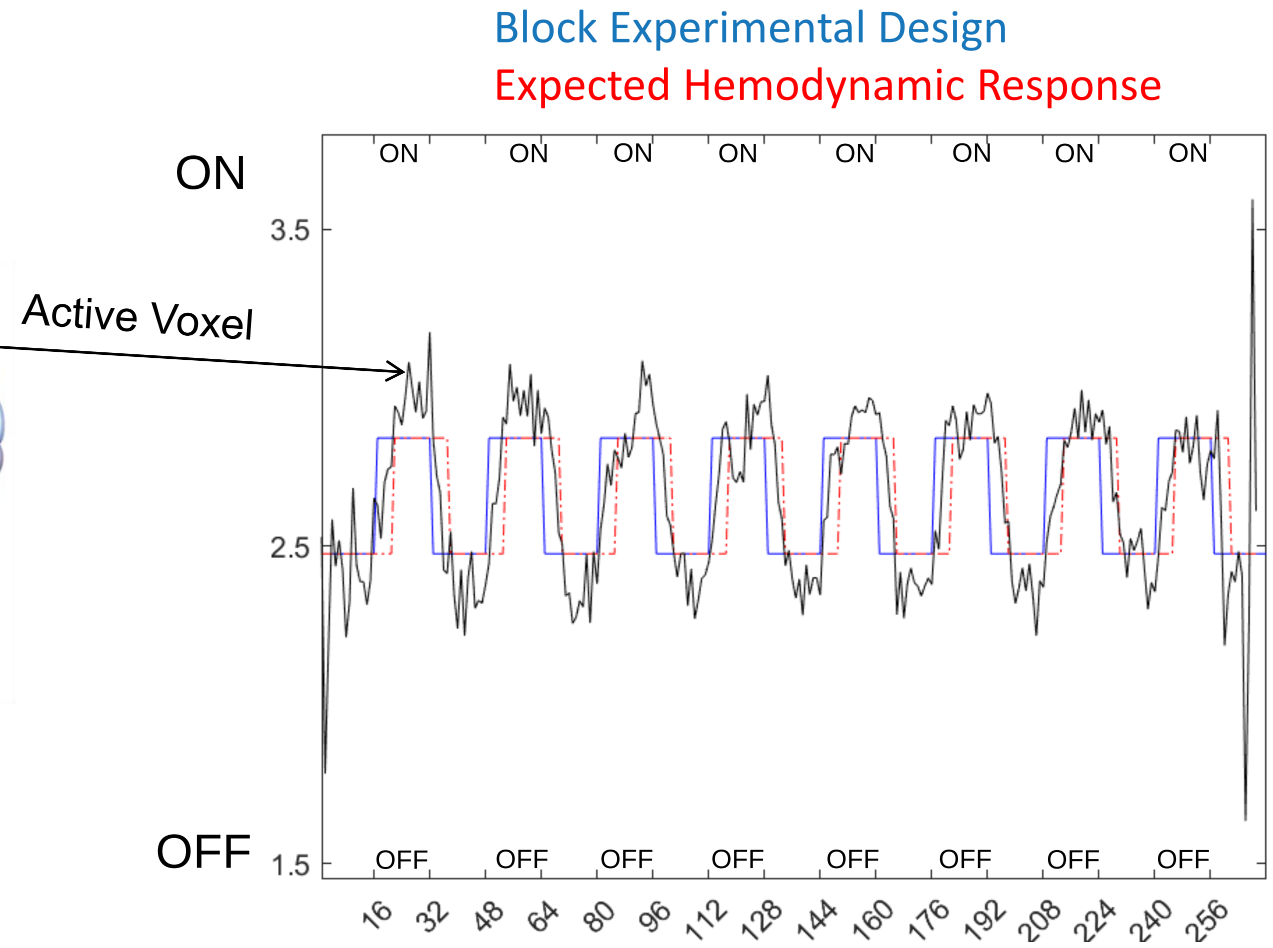
Background of FMRI

A bilateral sequential finger-tapping experiment was performed in a block design with 16-s off (no tap) followed by eight epochs of 16-s on (tap) and 16-s off (no-tap).

Tap Fingers

during blue on
expect red on





Rowe DB, Logan BR. A complex way to compute fMRI activation. Neuroimage 23(3):1078-1092 (2004).

Estimating Mean and Variance

In Chapter 2 of the textbook, we discussed measures of central tendency and measures of variability.

Mean, median, mode, deviation, z-score, variance, standard deviation, ….

Estimating Mean and Variance

Remember this slide?

Class 02 Lecture

Equation 2.1 p. 63

Marquette University MATH 1700/MATH1700H

2: Descriptive Analysis and Single Variable Data

2.3 Measures of Central Tendency

Sample Mean: The usual average you are familiar with.
Represented by $\bar{x}$ called "x-*bar.*" p. 63

Simply add up all the values and divide by the number values.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Remember the sigma notation we reviewed?

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \ldots + x_n$$

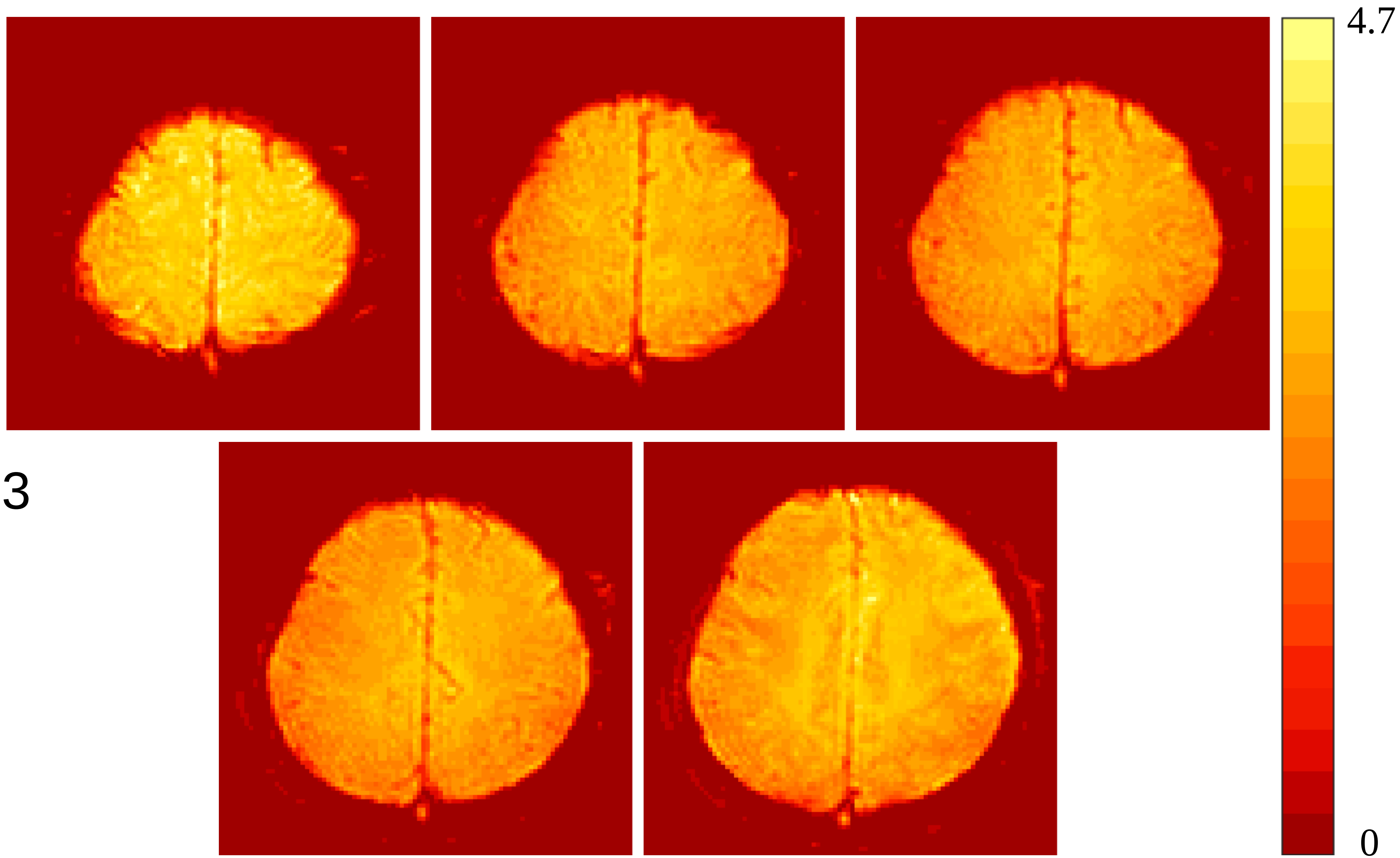
Round-off Rule: When rounding a number, let's keep one more decimal place than the original numbers.

Rowe, D.B. 28

Estimating Mean and Variance

Chapter 2

Sample Mean



Equation 2.1 p. 63

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

In each voxel.

Estimating Mean and Variance

Remember this slide?

Class 02 Lecture

Equation 2.9 p. 77

2: Descriptive Analysis and Single Variable Data

2.4 Measures of Dispersion

Sample Variance: The mean of the squared deviations using n-1 as a divisor. p. 75

There are two equivalent formulas that can be used.

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \qquad (2.5)$$

and

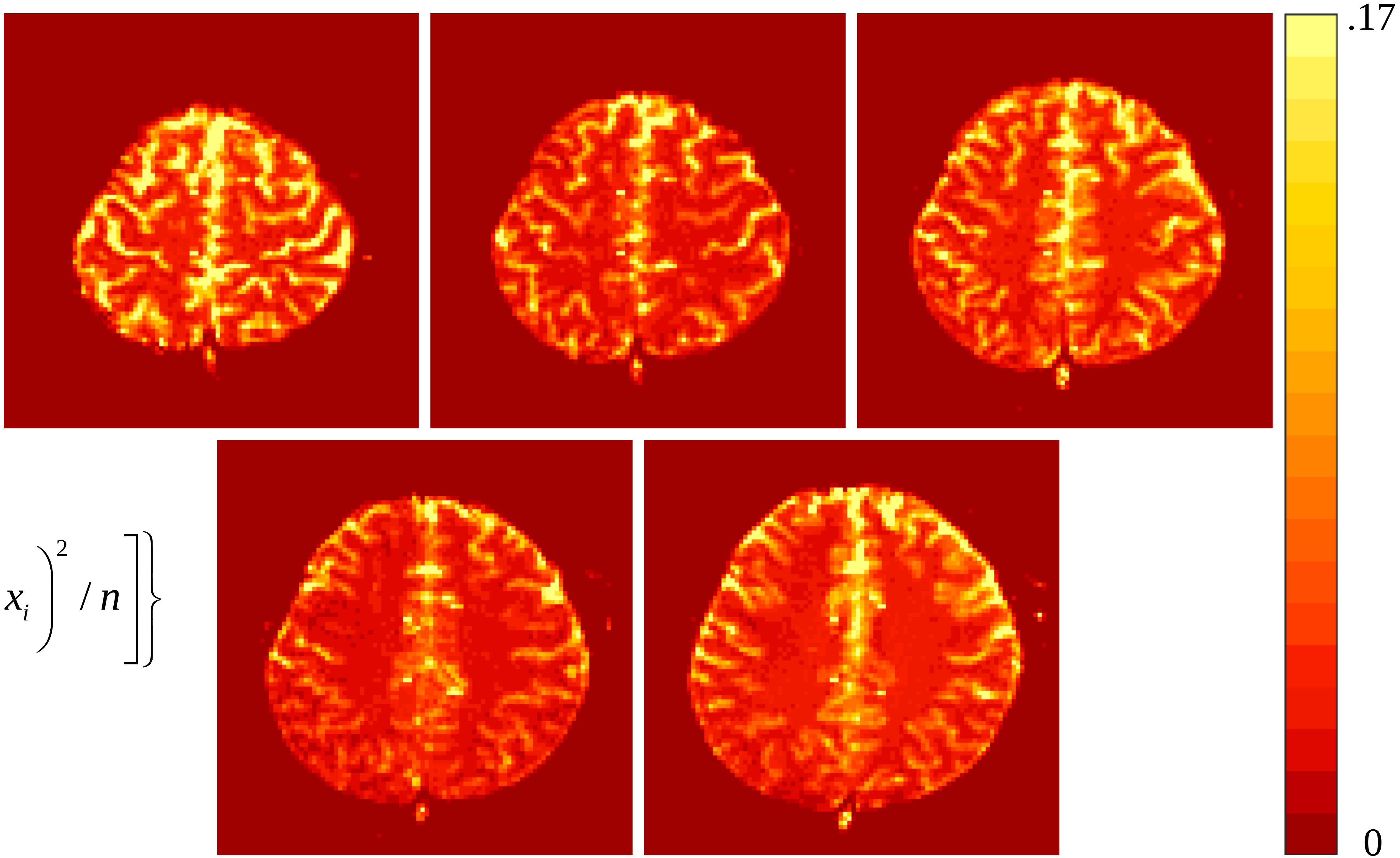
$$s^2 = \frac{1}{n-1}\left\{\sum_{i=1}^{n} x_i^2 - \left[\left(\sum_{i=1}^{n} x_i\right)^2 / n\right]\right\} \qquad (2.9)$$

where x_i is i^{th} data value, $\bar{x}$ is sample mean, n is sample size.

Estimating Mean and Variance

Chapter 2

Sample Variance



Equation 2. p. 76

$$s^2 = \frac{1}{n-1}\left\{\sum_{i=1}^{n} x_i^2 - \left[\left(\sum_{i=1}^{n} x_i\right)^2 / n\right]\right\}$$

In each voxel.

Hypothesis Testing on Mean

In Chapter 9 of the textbook, we discussed hypothesis testing on the mean μ assuming we don't know σ^2.

H_0: $\mu \geq \mu_0$ vs. H_a: $\mu < \mu_0$

H_0: $\mu \leq \mu_0$ vs. H_a: $\mu > \mu_0$

H_0: $\mu = \mu_0$ vs. H_a: $\mu \neq \mu_0$

Hypothesis Testing on Mean

Remember this slide?

Class 17 Lecture

Equation 9.2 p.420

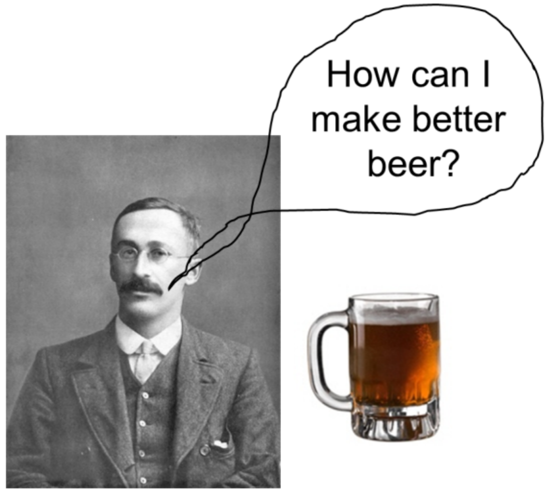
9: Inferences Involving One Population

9.1 Inference about the Mean μ (σ Unknown)

However, in real life, we never know σ for

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad (8.4)$$

so we would like to estimate σ by s, then use



$$t^* = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \quad . \quad (9.2)$$

But t^* does not have a standard normal distribution.

It has what is called a Student t-distribution. ⟵ Gosset Guinness Brewery

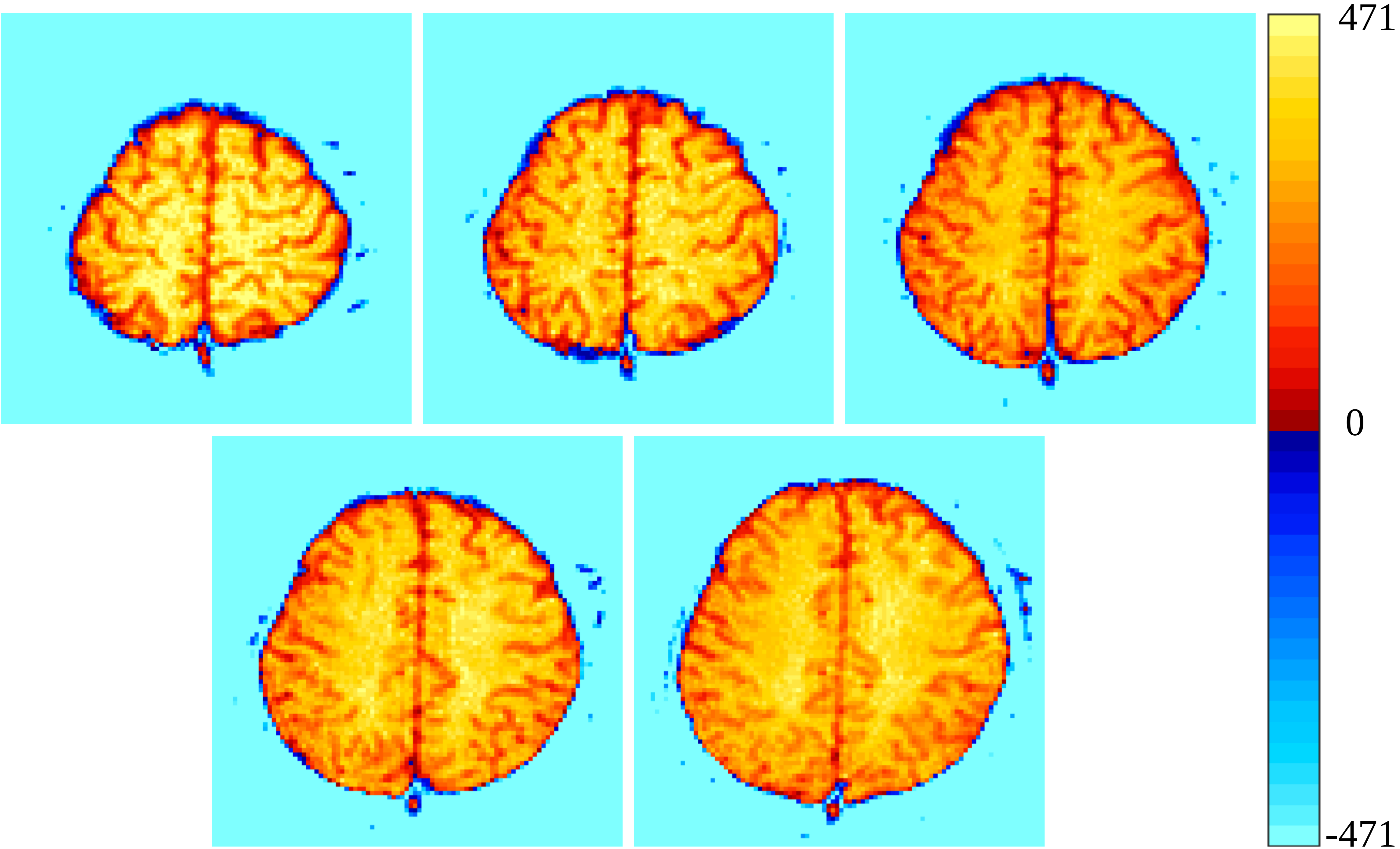
Hypothesis Testing on Mean

H_0: $\mu \leq 1.0$

vs.

H_a: $\mu > 1.0$

$$t^* = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$



In each voxel.

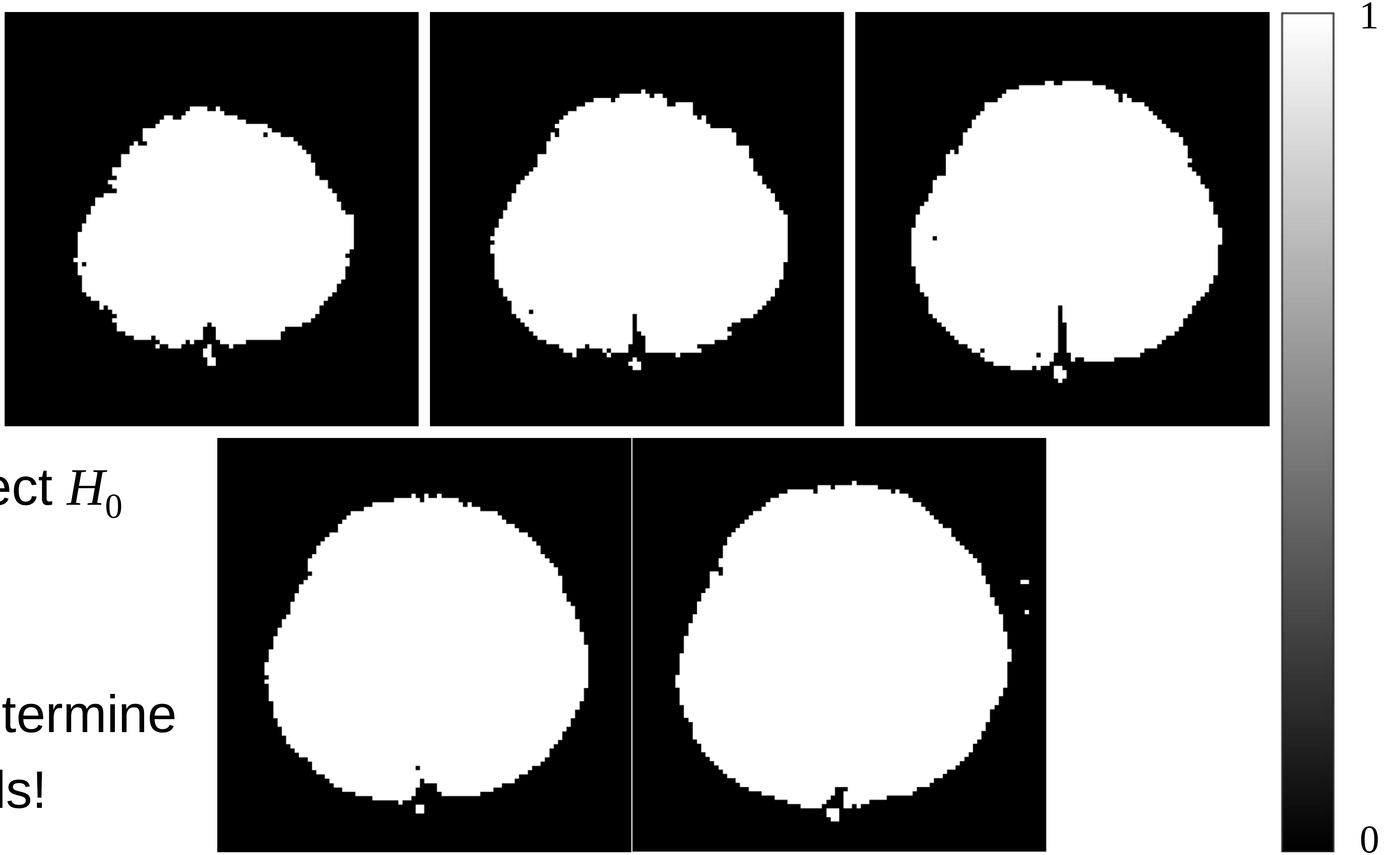
Hypothesis Testing on Mean

H_0: $\mu \leq 1.0$

vs.

H_a: $\mu > 1.0$

α=0.05, t_{crit}=1.65



Black Fail to Reject H_0

White Reject H_0

We can easily determine within brain voxels!

Hypothesis Testing on Variance

In Chapter 9 of the textbook, we discussed hypothesis testing on the variance σ^2.

H_0: $\sigma^2 \geq \sigma_0^2$ vs. H_a: $\sigma^2 < \sigma_0^2$

H_0: $\sigma^2 \leq \sigma_0^2$ vs. H_a: $\sigma^2 > \sigma_0^2$

H_0: $\sigma^2 = \sigma_0^2$ vs. H_a: $\sigma^2 \neq \sigma_0^2$

Hypothesis Testing on Variance

Remember this slide?

Class 19 Lecture

Equation 9.10. p. 456

9: Inferences Involving One Population

9.3 Inference about the Variance and Standard Deviation

Test Statistic for Variance (and Standard Deviation)

$$\chi^{2}* = \frac{(n-1)s^{2}}{\sigma_0^{2}}, \quad \text{with } df = n-1. \qquad (9.10)$$

sample variance (s^2); hypothesized population variance (σ_0^2)

Will also need critical values.

$$P\left(\chi^{2} > \chi^{2}(df,\alpha)\right) = \alpha$$

Table 8
Appendix B
Page 721

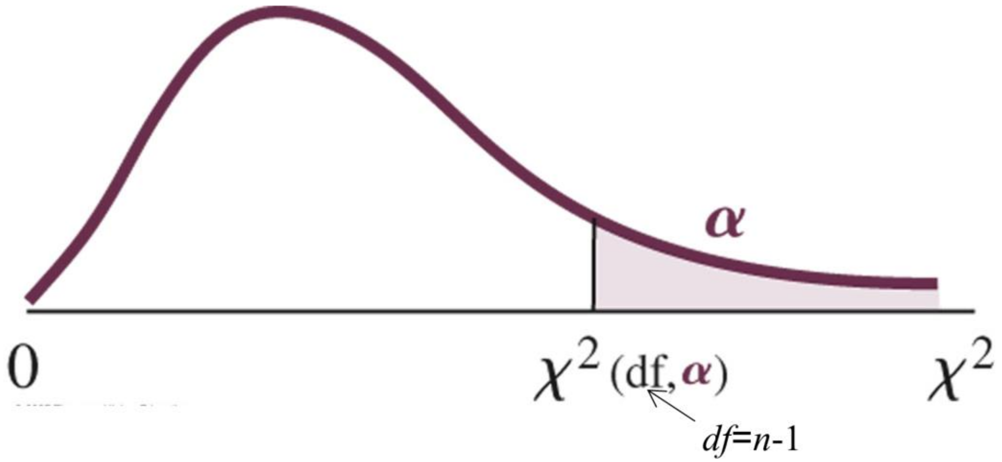


Figure from Johnson & Kuby, 2012.

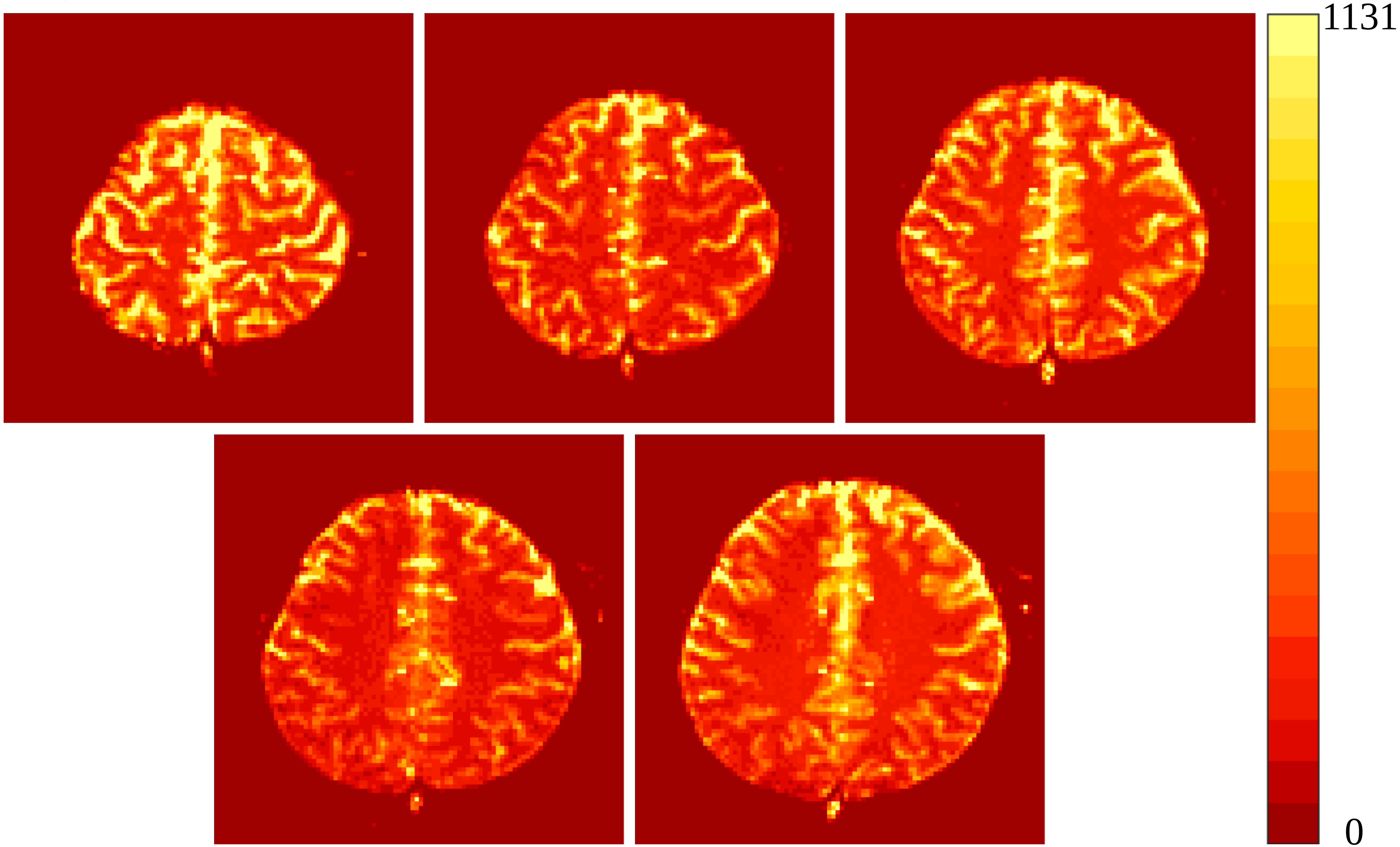
Hypothesis Testing on Variance

H_0: $\sigma^2 \leq 0.009$

vs.

H_a: $\sigma^2 > 0.009$

$$\chi^{2*} = \frac{(n-1)s^2}{\sigma_0^2}$$



In each voxel.

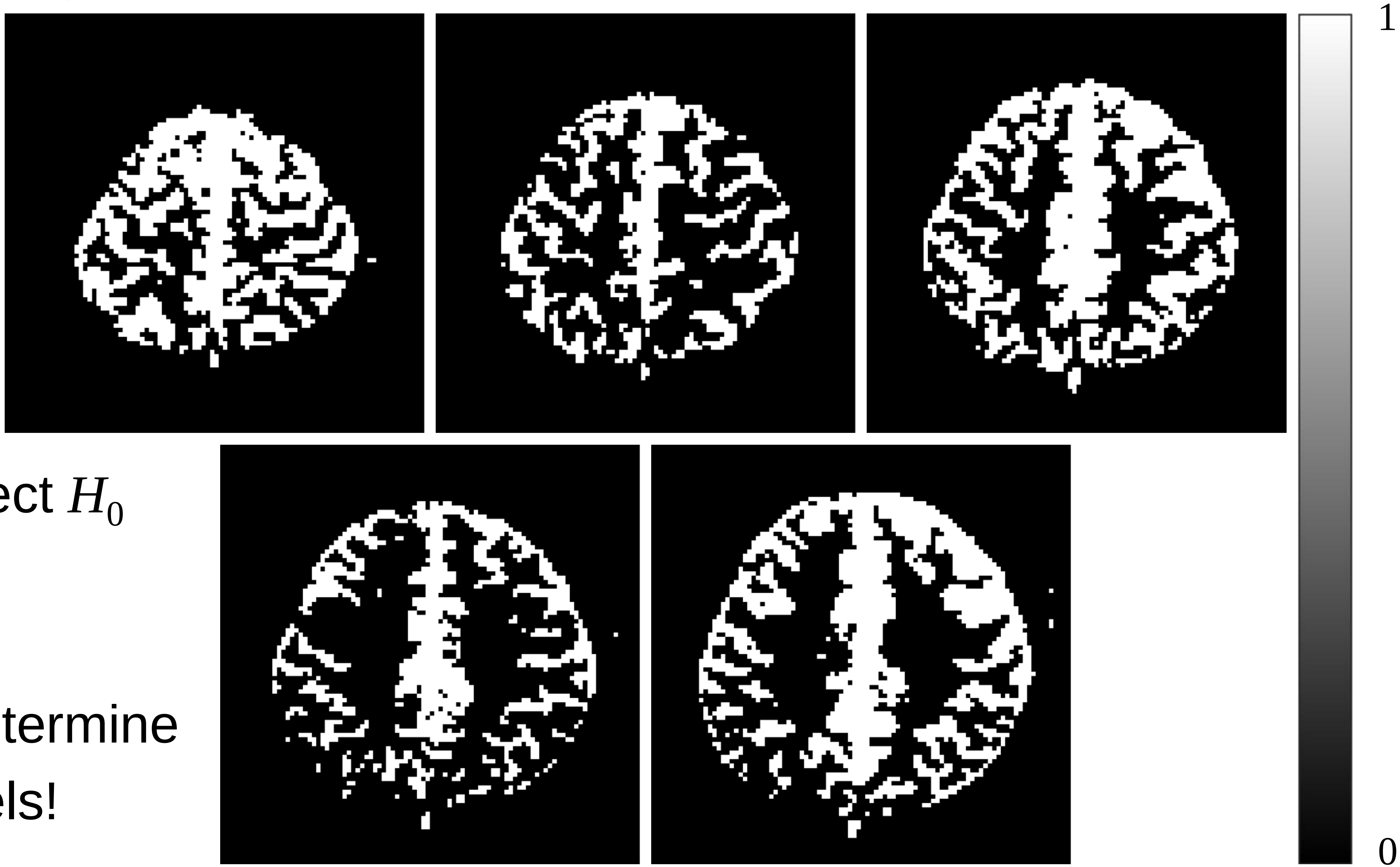
Hypothesis Testing on Variance

$H_0: \sigma^2 \leq 0.009$

vs.

$H_a: \sigma^2 > 0.009$

$\alpha = 0.05, \chi^2 crit = 307$



Black Fail to Reject H_0

White Reject H_0

We can easily determine

Grey matter voxels!

Hypothesis Testing on Variance

In Chapter 10 of the textbook, we discussed hypothesis testing on the difference in means $\mu_1 - \mu_2$.

H_0: $\mu_1 \geq \mu_2$ vs. H_a: $\mu_1 < \mu_2$

H_0: $\mu_1 \leq \mu_2$ vs. H_a: $\mu_1 > \mu_2$

H_0: $\mu_1 = \mu_2$ vs. H_a: $\mu_1 \neq \mu_2$

Hypothesis Testing on Difference in Means

10: Inferences Involving Two Populations

10.3 Inference for Mean Difference Two Independent Samples

Hypothesis Testing Procedure

We can test for differences in the population means:

H_0: $\mu_1 \geq \mu_2$ vs. H_a: $\mu_1 < \mu_2$ $\rightarrow$ H_0: $\mu_1 - \mu_2 \geq 0$ vs. H_a: $\mu_1 - \mu_2 < 0$

H_0: $\mu_1 \leq \mu_2$ vs. H_a: $\mu_1 > \mu_2$ $\rightarrow$ H_0: $\mu_1 - \mu_2 \leq 0$ vs. H_a: $\mu_1 - \mu_2 > 0$

H_0: $\mu_1 = \mu_2$ vs. H_a: $\mu_1 \neq \mu_2$ $\rightarrow$ H_0: $\mu_1 - \mu_2 = 0$ vs. H_a: $\mu_1 - \mu_2 \neq 0$

Hypothesis Testing on Difference in Mean

Remember this slide?

Class 19 Lecture

Equation 9.10. p. 456

10: Inferences Involving Two Populations

10.3 Inference for Mean Difference Two Independent Samples

Hypothesis Testing Procedure

With σ_1 and σ_2 unknown, the test statistic for $\mu_1 - \mu_2$ is:

Test Statistic for Mean Difference (Independent Samples)

$$t^* = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_{0,1} - \mu_{0,2})}{\sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)}}$$

Next larger number than

$$df = \left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2 \Bigg/ \left(\frac{\left(s_1^2/n_1\right)^2}{n_1 - 1} + \frac{\left(s_2^2/n_2\right)^2}{n_2 - 1}\right)$$

If not using a computer program.

where df is either calculated or smaller of df_1, or df_2 (10.9)

Actually, this is for $\sigma_1 \neq \sigma_2$.

If using a computer program.

Go through the same five hypothesis testing steps.

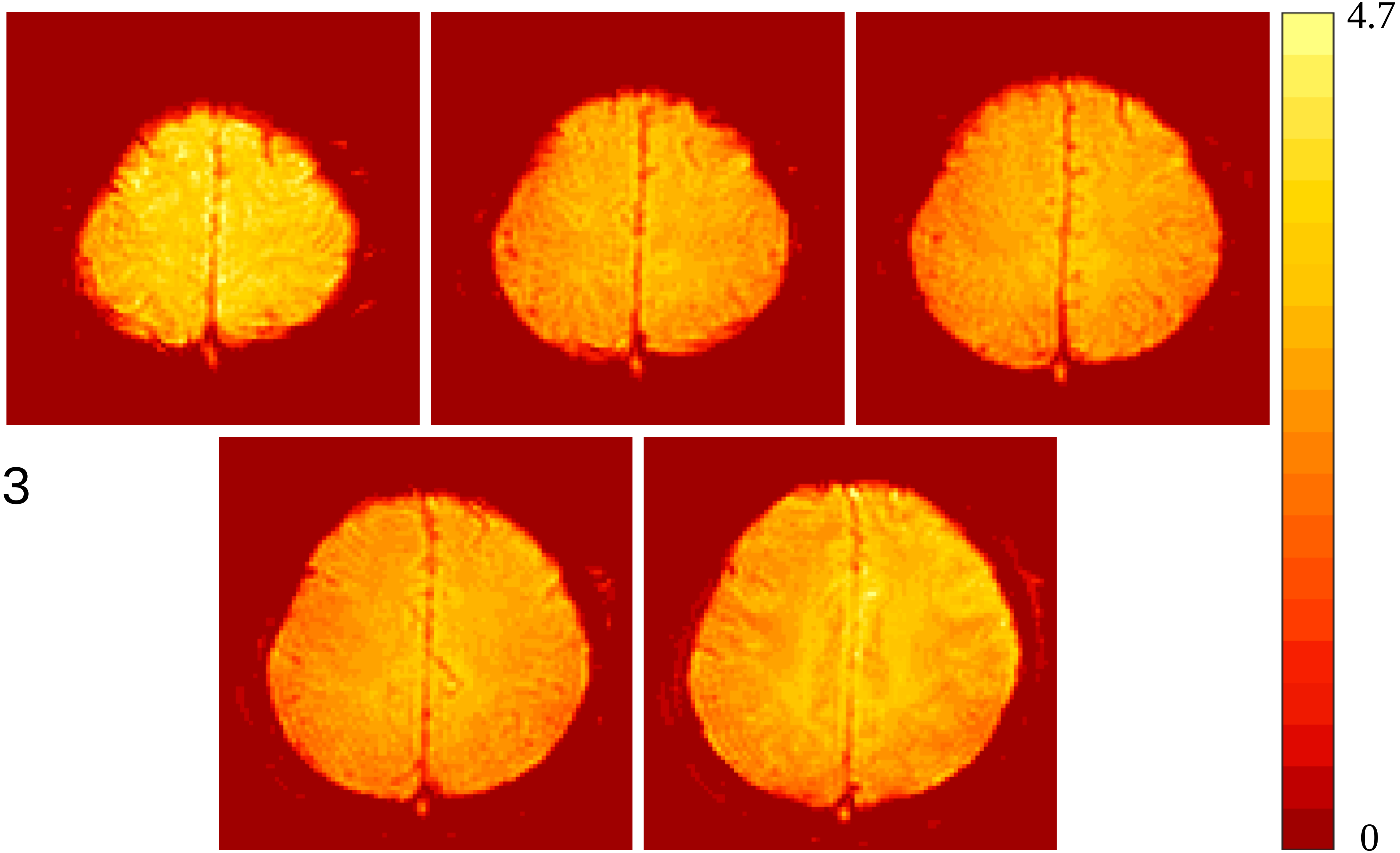
Need normal populations to use t critical values.

Hypothesis Testing on Difference in Mean

Chapter 10

Sample Mean 1

(during task)



Equation 2.1 p. 63

$$\bar{x}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} x_{1i}$$

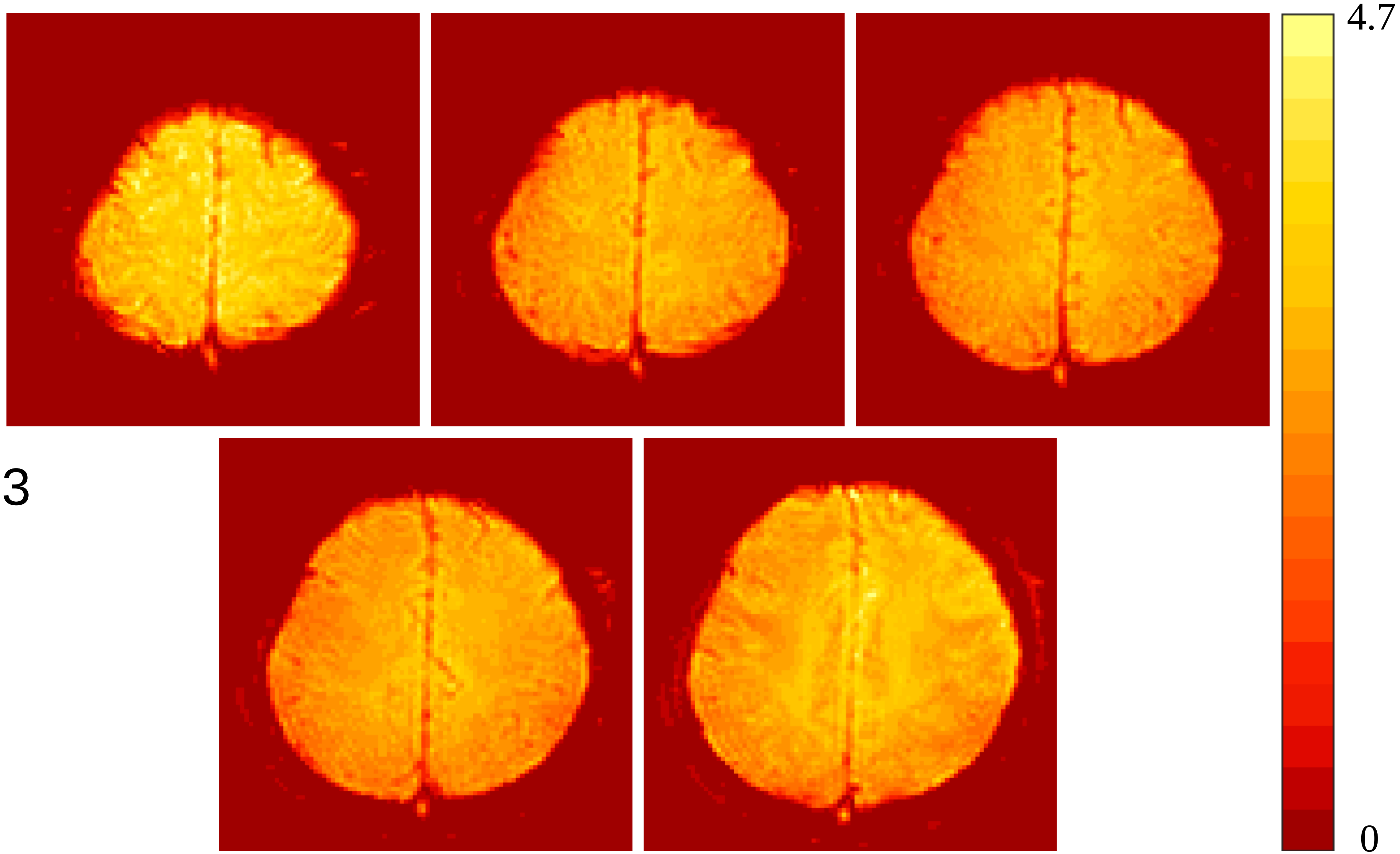
In each voxel.

Hypothesis Testing on Difference in Mean

Chapter 10

Sample Mean 2

(during nontask)



Equation 2.1 p. 63

$$\bar{x}_2 = \frac{1}{n_2}\sum_{i=1}^{n_2} x_{2i}$$

In each voxel.

Hypothesis Testing on Difference in Mean

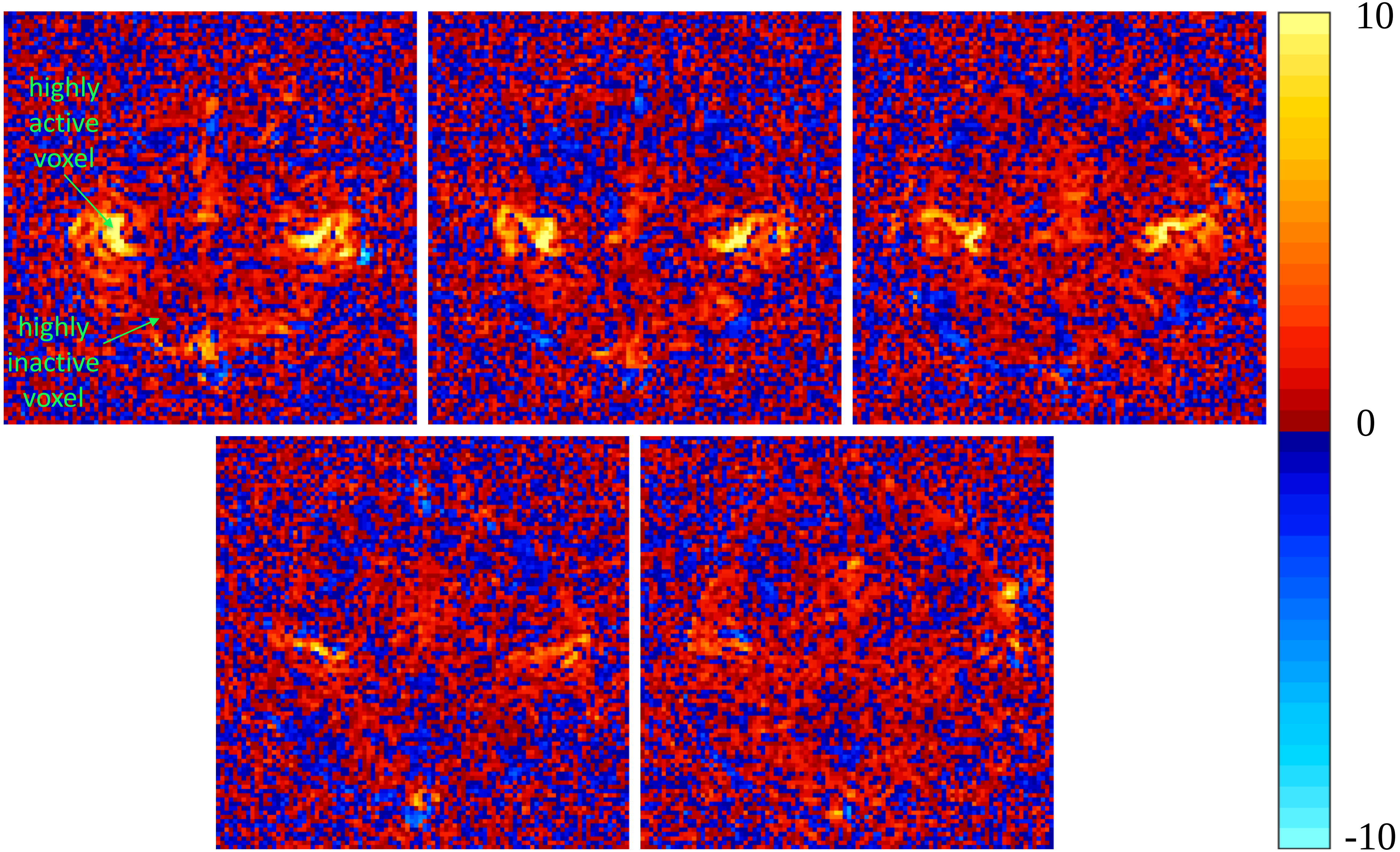
$H_0: \mu_1 \leq \mu_2$

vs.

$H_a: \mu_1 > \mu_2$

$$t^* = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_{0,1} - \mu_{0,2})}{\sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)}}$$

In each voxel.



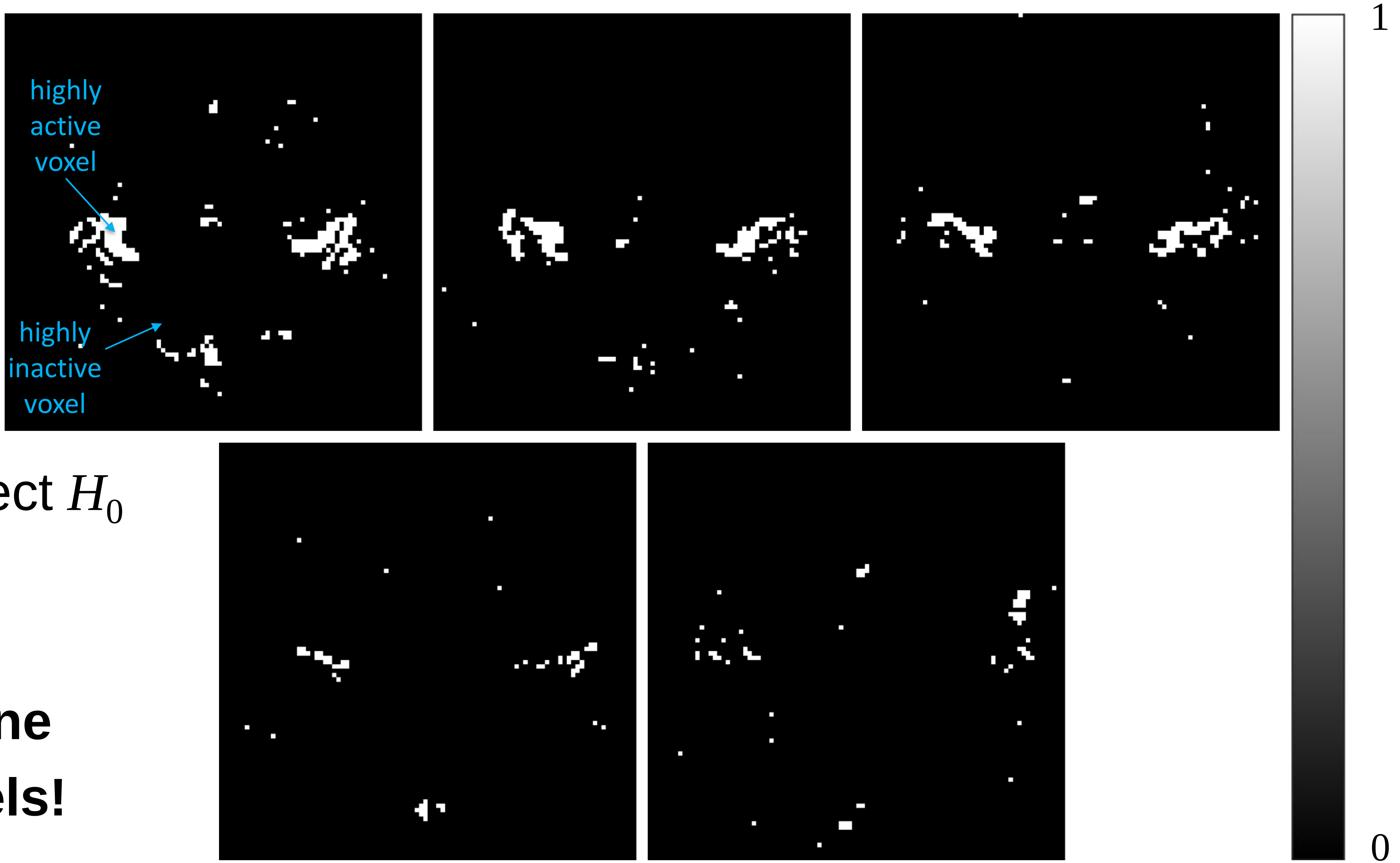
Hypothesis Testing on Difference in Mean

H_0: $\mu_1 \leq \mu_2$

vs.

H_a: $\mu_1 > \mu_2$

α=.0001, t_{crit}=3.83



Black Fail to Reject H_0

White Reject H_0

We can determine task active voxels!

Hypothesis Testing on Difference in Mean

$$H_0\!: \mu_1 \le \mu_2$$

vs.

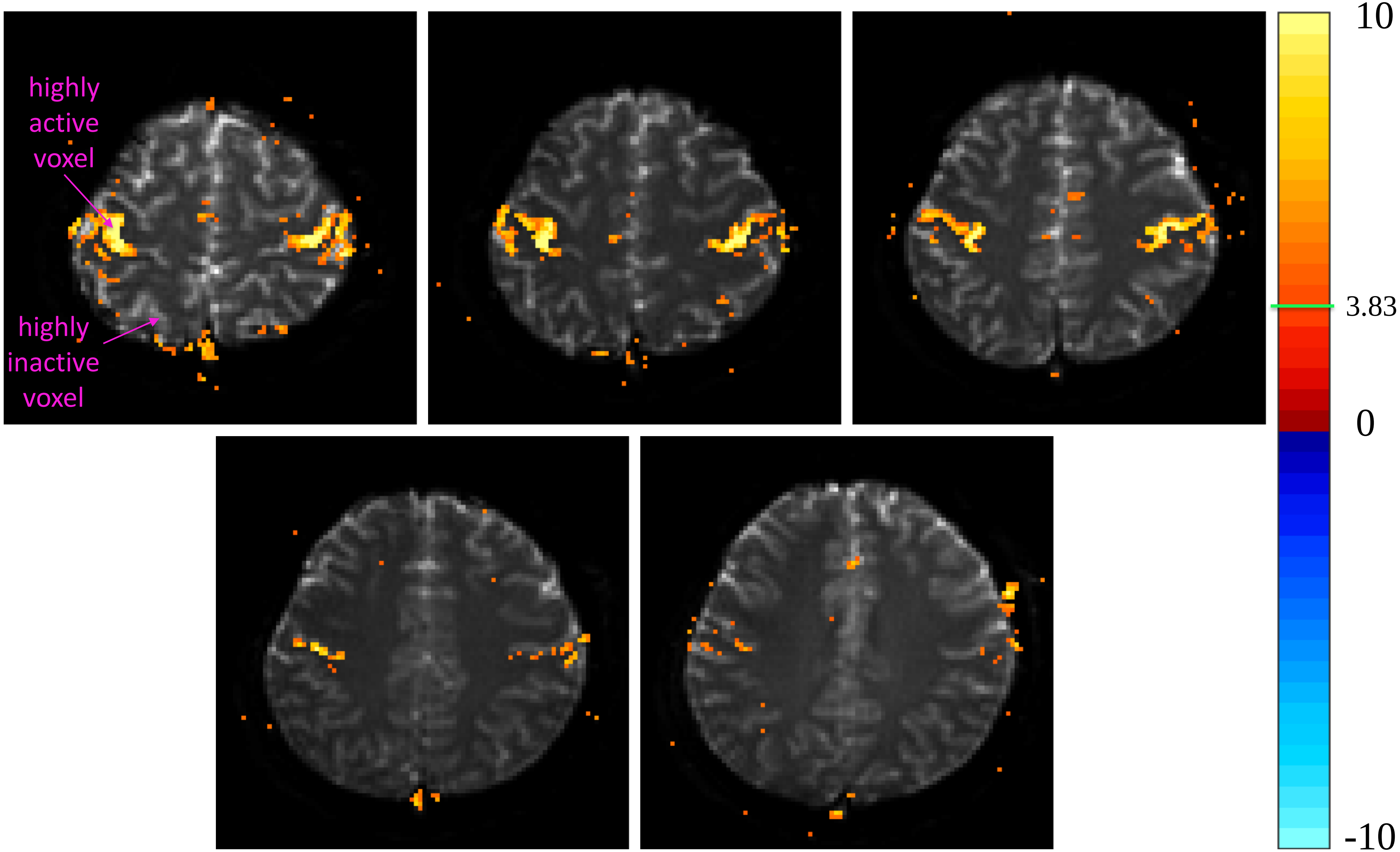
$$H_a\!: \mu_1 > \mu_2$$

$$t^* = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_{0,1} - \mu_{0,2})}{\sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)}}$$

$\alpha = .0001$

Superimpose.

In each voxel.



Discussion

The statistical methods that we've learned in class are very powerful. There are many important applications where they can be used.

This is only one possible important application.

You are really learning great stuff and there is a reason you are in this class.

Discussion

Questions?

Homework

1. Perform the hypothesis test for differences in population means

$H_0\!: \mu_1 \leq \mu_2$ vs. $H_a\!: \mu_1 > \mu_2$

$$t^* = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)}}$$

$\alpha = 0.0001$

for the one active and one inactive voxels.