

# Class 21

Daniel B. Rowe, Ph.D.

Department of Mathematical and Statistical Sciences



# Agenda:

**Recap Chapter 10.1 and 10.2**

**Lecture Chapter 10.3**

# Recap Chapter 10.1 and 10.2

# 10: Inferences Involving Two Populations

## 10.2 Inference for Mean Difference Two Dependent Samples

We form a paired difference from the data

### Paired Difference

$$d = x_1 - x_2 \quad (10.1)$$

This means that we are subtracting the sample value from population 2 from the sample value from population 1.

# 10: Inferences Involving Two Populations

## 10.2 Inference for Mean Difference Two Dependent Samples

So if the  $d_i$ 's are approximately normally distributed

with a mean of  $\mu_d$  and a standard deviation of  $\sigma_d$ , then

$$\bar{d} = \frac{1}{n} \sum_{i=1}^n d_i \text{ is normally distributed (recall CLT)}$$

with a mean  $\mu_{\bar{d}} = \mu_d$ , and standard deviation  $\sigma_{\bar{d}} = \frac{\sigma_d}{\sqrt{n}}$ .

# 10: Inferences Involving Two Populations

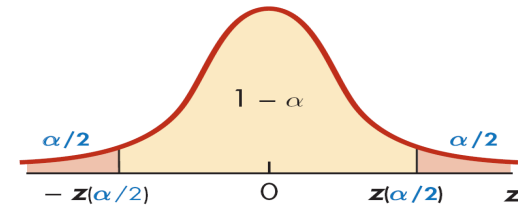
## 10.2 Inference for Mean Difference Two Dependent Samples

This would allow us to form a  $z$  statistic for the mean of

differences  $\bar{d}$ ,  $z = \frac{\bar{d} - \mu_d}{\sigma_d / \sqrt{n}}$  with a standard normal distribution.

← assuming known

We can then look up probabilities in the table,



find critical values  $z(\alpha/2)$ , construct confidence intervals

and test hypotheses using  $z^* = \frac{\bar{d} - \mu_{0d}}{\sigma_d / \sqrt{n}}$ .

$\bar{d} \pm z(\alpha/2) \frac{\sigma_d}{\sqrt{n}}$

Figure from Johnson & Kuby, 2012.

# 10: Inferences Involving Two Populations

## 10.2 Inference for Mean Difference Two Dependent Samples

However, as in Inferences for One Population, we never know the true value of  $\sigma_d$ . So we estimate it with sample

standard deviation  $s_d$ . This changes  $z = \frac{\bar{d} - \mu_d}{\sigma_d / \sqrt{n}}$

to  $t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}}$  and the distribution from standard normal

to Student  $t$  with  $df=n-1$  where  $s_d^2 = \frac{1}{n-1} \sum_{i=1}^n (d_i - \bar{d})^2$ .

# 10: Inferences Involving Two Populations

## 10.2 Inference for Mean Difference Two Dependent Samples Confidence Interval Procedure

With  $\sigma_d$  unknown, a  $1-\alpha$  confidence interval for  $\mu_d$  is:

### Confidence Interval for Mean Difference (Dependent Samples)

$$\bar{d} - t(df, \alpha / 2) \frac{s_d}{\sqrt{n}} \quad \text{to} \quad \bar{d} + t(df, \alpha / 2) \frac{s_d}{\sqrt{n}} \quad \text{where } df=n-1 \quad (10.2)$$

$$\bar{d} = \frac{1}{n} \sum_{i=1}^n d_i \quad (10.3)$$

$$s_d^2 = \frac{1}{n-1} \sum_{i=1}^n (d_i - \bar{d})^2 \quad (10.4)$$



# 10: Inferences Involving Two Populations

## 10.2 Inference for Mean Difference Two Dependent Samples

Car	1	2	3	4	5	6
Brand A	125	64	94	38	90	106
Brand B	133	65	103	37	102	115

### Example:

Construct a 95% CI for mean difference in Brand B – A tire wear.

$d_i$ 's: 8, 1, 9, -1, 12, 9

$$n = 6$$

$$df = 5$$

$$t(df, \alpha / 2) = 2.57$$

$$\bar{d} = 6.3$$

$$\alpha = 0.05$$

$$s_d = 5.1$$

$$\bar{d} \pm t(df, \alpha / 2) \frac{s_d}{\sqrt{n}} \longrightarrow (0.090, 11.7)$$

$$\bar{d} = \frac{1}{n} \sum_{i=1}^n d_i$$

$$s_d^2 = \frac{1}{n-1} \sum_{i=1}^n (d_i - \bar{d})^2$$

Figure from Johnson & Kubly, 2012.

# 10: Inferences Involving Two Populations

## 10.2 Inference for Mean Difference Two Dependent Samples Hypothesis Testing Procedure

We can test for differences in the population means:

$$H_0: \mu_1 \geq \mu_2 \text{ vs. } H_a: \mu_1 < \mu_2 \quad \rightarrow \quad H_0: \mu_1 - \mu_2 \geq 0 \text{ vs. } H_a: \mu_1 - \mu_2 < 0$$

$$H_0: \mu_1 \leq \mu_2 \text{ vs. } H_a: \mu_1 > \mu_2 \quad \rightarrow \quad H_0: \mu_1 - \mu_2 \leq 0 \text{ vs. } H_a: \mu_1 - \mu_2 > 0$$

$$H_0: \mu_1 = \mu_2 \text{ vs. } H_a: \mu_1 \neq \mu_2 \quad \rightarrow \quad H_0: \mu_1 - \mu_2 = 0 \text{ vs. } H_a: \mu_1 - \mu_2 \neq 0$$

$$\mu_d = \mu_1 - \mu_2 \quad \rightarrow \quad H_0: \mu_d \geq 0 \text{ vs. } H_a: \mu_d < 0$$

$$(\mu_d = \mu_{\text{before}} - \mu_{\text{after}}) \quad H_0: \mu_d \leq 0 \text{ vs. } H_a: \mu_d > 0$$

$$H_0: \mu_d = 0 \text{ vs. } H_a: \mu_d \neq 0$$

# 10: Inferences Involving Two Populations

## 10.2 Inference for Mean Difference Two Dependent Samples Hypothesis Testing Procedure

With  $\sigma_d$  unknown, the test statistic for  $\mu_d$  is:

### Test Statistic for Mean Difference (Dependent Samples)

$$t^* = \frac{\bar{d} - \mu_{d0}}{s_d / \sqrt{n}} \quad \text{where } df = n - 1 \quad (10.5)$$

Go through the same five hypothesis testing steps.

# 9: Inferences Involving One Population

## 9.2 Inference about the Binomial Probability of Success

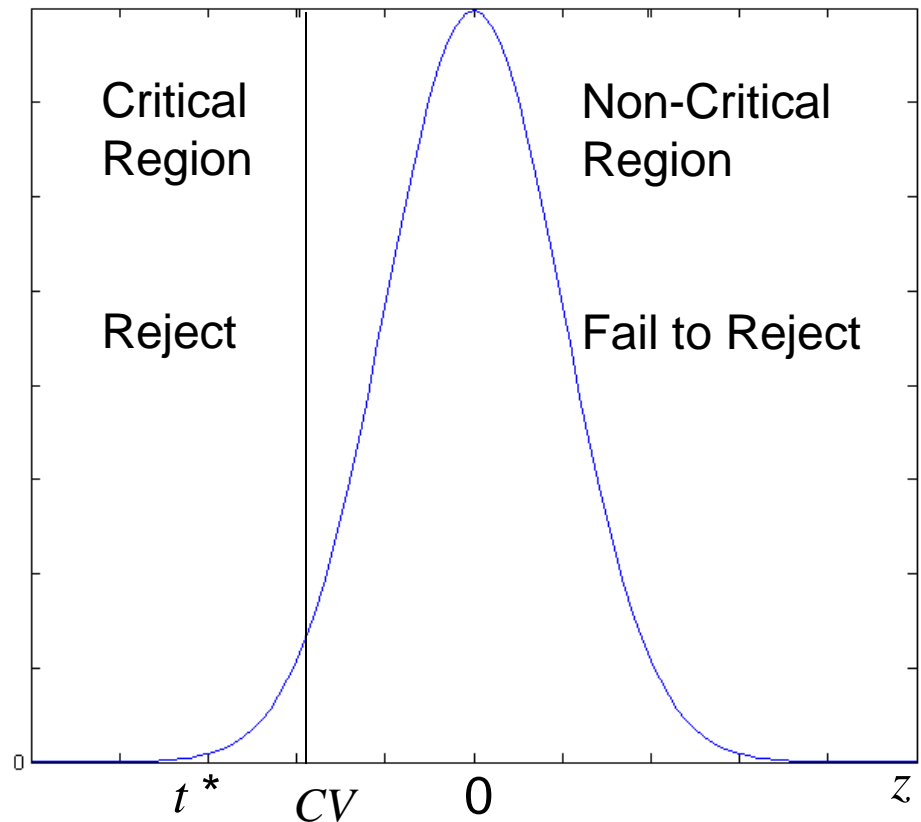
There are three possible hypothesis pairs for the difference in means.

$$H_0: \mu_d \geq \mu_{d0} \text{ vs. } H_a: \mu_d < \mu_{d0}$$

Reject  $H_0$  if  $t^*$  is less than

$$t^* = \frac{\bar{d} - \mu_{d0}}{s_d / \sqrt{n}} \quad -t(df, \alpha)$$

data indicates  $\mu_d < \mu_{d0}$   
because  $\bar{d}$  is “a lot”  
smaller than  $\mu_{d0}$



# 9: Inferences Involving One Population

## 9.2 Inference about the Binomial Probability of Success

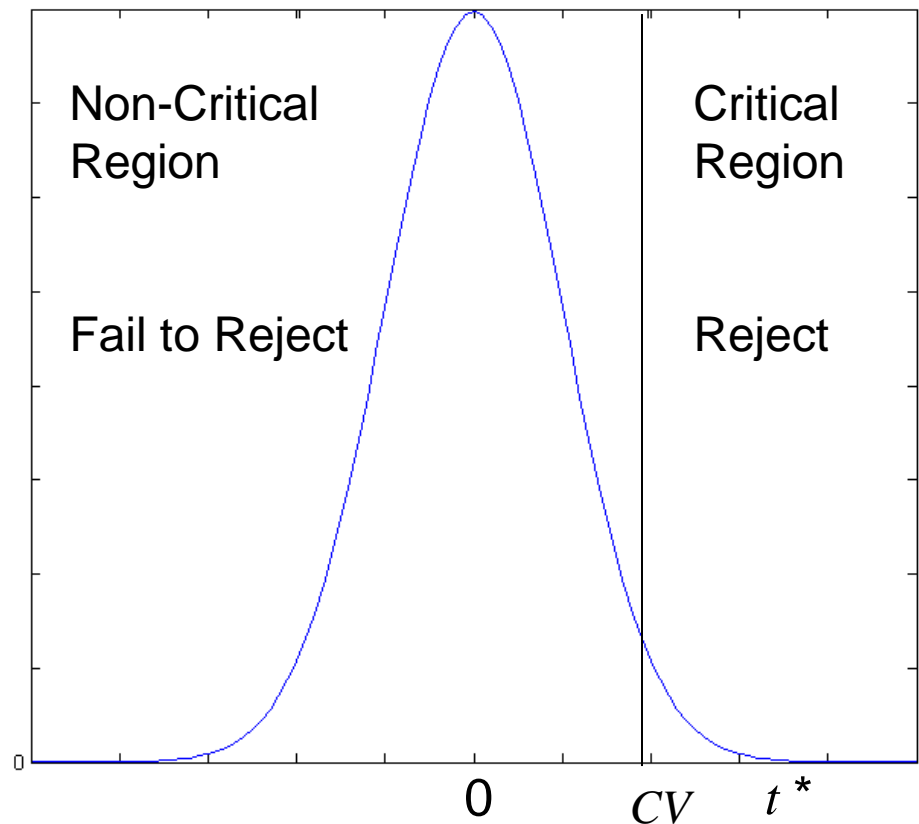
There are three possible hypothesis pairs for the difference in means.

$$H_0: \mu_d \leq \mu_{d0} \text{ vs. } H_a: \mu_d > \mu_{d0}$$

Reject  $H_0$  if  $t$  is greater than

$$t^* = \frac{\bar{d} - \mu_{d0}}{s_d / \sqrt{n}} \quad t(df, \alpha)$$

data indicates  $\mu_d > \mu_{d0}$   
because  $\bar{d}$  is “a lot”  
smaller than  $\mu_{d0}$



# 9: Inferences Involving One Population

## 9.2 Inference about the Binomial Probability of Success

There are three possible hypothesis pairs for the difference in means.

$$H_0: \mu_d = \mu_{d0} \text{ vs. } H_a: \mu_d \neq \mu_{d0}$$

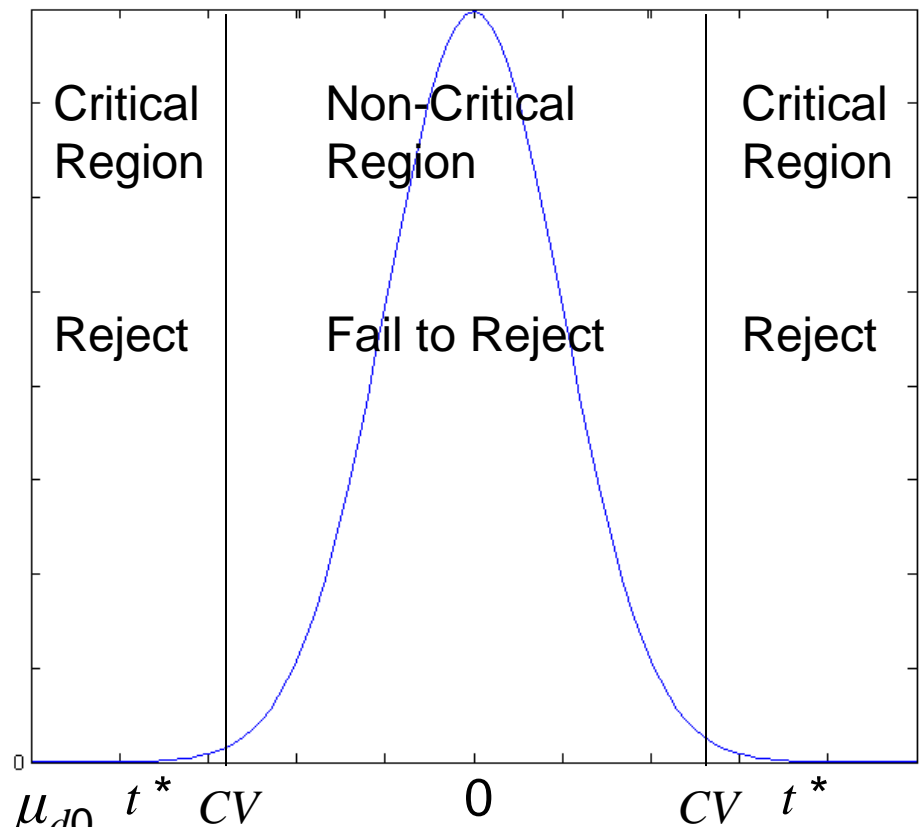
Reject  $H_0$  if  $\frac{\bar{d} - \mu_{d0}}{s_d / \sqrt{n}}$  is less than

$$-t^* = \frac{\bar{d} - \mu_{d0}}{s_d / \sqrt{n}} < -t(df, \alpha / 2)$$

or if  $\frac{\bar{d} - \mu_{d0}}{s_d / \sqrt{n}}$  is greater than

$$t^* = \frac{\bar{d} - \mu_{d0}}{s_d / \sqrt{n}} > t(df, \alpha / 2)$$

data indicates  $\mu_d \neq \mu_{d0}$ ,  $\bar{d}$  far from  $\mu_{d0}$



# 10: Inferences Involving Two Populations

## 10.2 Inference for Mean Difference Two Dependent Samples

$$n = 6 \quad 8, 1, 9, -1, 12, 9$$

Car	1	2	3	4	5	6
Brand A	125	64	94	38	90	106
Brand B	133	65	103	37	102	115

### Example:

Test mean difference of Brand B minus Brand A is zero.

Step 1  $H_0: \mu_d = 0$  vs.  $H_a: \mu_d \neq 0$

Step 2

$$df = 5 \quad t^* = \frac{\bar{d} - \mu_{d0}}{s_d / \sqrt{n}}$$

$$\alpha = .05$$

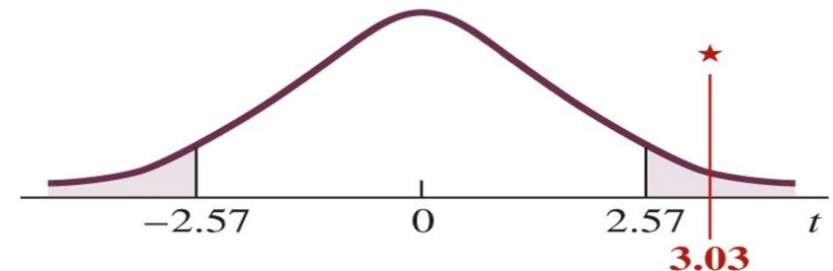
Step 3  $\bar{d} = 6.3$   $t^* = \frac{6.3 - 0}{5.1 / \sqrt{6}} = 3.03$

$$s_d = 5.1$$

Step 4  $t(df, \alpha / 2) = 2.57$

Step 5 Since  $t^* > t(df, \alpha/2)$ , reject  $H_0$

different	same	different
-----------	------	-----------



Conclusion: Significant difference in tread wear at .05 level.

Figures from Johnson & Kuby, 2012.

# Chapter 10: Inferences Involving Two Populations

Questions?

Homework: Read Chapter 10.1-10.2

WebAssign

Chapter 10 #13, 15, 23, 25, 29, 31, 35



# Chapter 10: Inference Involving Two Populations

Daniel B. Rowe, Ph.D.

Department of Mathematical and Statistical Sciences



# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples

### Background

We said from SDSM that  $\text{mean}(\bar{x}) = \mu$  and  $\text{variance}(\bar{x}) = \frac{\sigma^2}{n}$ .

We are often interested in comparisons between means  $\bar{x}_1 - \bar{x}_2$ .

There's a rule that says that if  $\bar{x}_1$  and  $\bar{x}_2$  have means  $\mu_1$  and  $\mu_2$ ,

and variances  $\sigma_1^2$  and  $\sigma_2^2$ ,

then  $\text{mean}(\bar{x}_1 - \bar{x}_2) = \mu_1 - \mu_2$

and  $\text{variance}(\bar{x}_1 - \bar{x}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} \longrightarrow \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ .

if  $x_1$  &  $x_2$  independent

Variances add not standard deviations.

# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples Means Using Two Independent Samples

If two populations are independent we can construct confidence intervals and test hypotheses for the difference in their means.

If independent samples of sizes  $n_1$  and  $n_2$  are drawn ... with means  $\mu_1$  and  $\mu_2$  and variances  $\sigma_1^2$  and  $\sigma_2^2$ , then the sampling distribution of  $\bar{x}_1 - \bar{x}_2$  ... has

1. mean  $\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2$  and

2. standard error  $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\left(\frac{\sigma_1^2}{n_1}\right) + \left(\frac{\sigma_2^2}{n_2}\right)}$

If both pops. are normal, then  $\bar{x}_1 - \bar{x}_2$  is normal . (10.6)

Actually the CLT works here for  $\bar{x}$ 's.

# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples

However, the true population variances are never truly known

so we estimate  $\sigma_1^2$  and  $\sigma_2^2$  by  $s_1^2$  and  $s_2^2$  and the

standard error

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\left(\frac{\sigma_1^2}{n_1}\right) + \left(\frac{\sigma_2^2}{n_2}\right)} \quad (10.6)$$

by

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)} . \quad (10.7)$$

# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples Confidence Interval Procedure

With  $\sigma_1$  and  $\sigma_2$  unknown, a  $1-\alpha$  confidence interval for  $\mu_1 - \mu_2$  is:

### Confidence Interval for Mean Difference (Independent Samples)

$$(\bar{x}_1 - \bar{x}_2) - t(df, \alpha / 2) \sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)} \quad \text{to} \quad (\bar{x}_1 - \bar{x}_2) + t(df, \alpha / 2) \sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)}$$

where  $df$  is either calculated or smaller of  $df_1$ , or  $df_2$  (10.8)

Actually, this is for  $\sigma_1 \neq \sigma_2$ .

Next larger number than

$$df = \left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2 \bigg/ \left(\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}\right)$$

If using a computer program.

If not using a computer program.

Need normal populations to use  $t$  critical values.

# 10: Inferences Involving Two Populations

## 10.3 Inference Mean Difference Confidence Interval

Sample	Number	Mean	Standard Deviation
Female ( <i>f</i> )	$n_1 = 20$	$\bar{x}_f = 63.8$	$s_f = 2.18$
Male ( <i>m</i> )	$n_2 = 30$	$\bar{x}_m = 69.8$	$s_m = 1.92$

### Example:

Interested in difference in mean heights between men and women. The heights of 20 females and 30 males is measured. Construct a 95% confidence interval for  $\mu_m - \mu_f$ ,  $\sigma_m$  &  $\sigma_f$  unknown

$$(\bar{x}_m - \bar{x}_f) \pm t(df, \alpha / 2) \sqrt{\left(\frac{s_m^2}{n_m}\right) + \left(\frac{s_f^2}{n_f}\right)}$$

$$\alpha = 0.05$$

$$t(19, .025) =$$

Figure from Johnson & Kuby, 2012.

# 10: Inferences Involving Two Populations

## 10.3 Inference Mean Difference Confidence Interval

Sample	Number	Mean	Standard Deviation
Female ( <i>f</i> )	$n_1 = 20$	$\bar{x}_f = 63.8$	$s_f = 2.18$
Male ( <i>m</i> )	$n_2 = 30$	$\bar{x}_m = 69.8$	$s_m = 1.92$

### Example:

Interested in difference in mean heights between men and women. The heights of 20 females and 30 males is measured. Construct a 95% confidence interval for  $\mu_m - \mu_f$ ,  $\sigma_m$  &  $\sigma_f$  unknown

$$(\bar{x}_m - \bar{x}_f) \pm t(df, \alpha / 2) \sqrt{\left(\frac{s_m^2}{n_m}\right) + \left(\frac{s_f^2}{n_f}\right)}$$

$$\alpha = 0.05$$

$$t(19, .025) = 2.09$$

$$(69.8 - 63.8) \pm 2.09 \sqrt{\left(\frac{(1.92)^2}{30}\right) + \left(\frac{(2.18)^2}{20}\right)}$$

therefore 4.75 to 7.25

Figure from Johnson & Kuby, 2012.

# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples Hypothesis Testing Procedure

We can test for differences in the population means:

$$H_0: \mu_1 \geq \mu_2 \text{ vs. } H_a: \mu_1 < \mu_2 \quad \rightarrow \quad H_0: \mu_1 - \mu_2 \geq 0 \text{ vs. } H_a: \mu_1 - \mu_2 < 0$$

$$H_0: \mu_1 \leq \mu_2 \text{ vs. } H_a: \mu_1 > \mu_2 \quad \rightarrow \quad H_0: \mu_1 - \mu_2 \leq 0 \text{ vs. } H_a: \mu_1 - \mu_2 > 0$$

$$H_0: \mu_1 = \mu_2 \text{ vs. } H_a: \mu_1 \neq \mu_2 \quad \rightarrow \quad H_0: \mu_1 - \mu_2 = 0 \text{ vs. } H_a: \mu_1 - \mu_2 \neq 0$$



# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples Hypothesis Testing Procedure

With  $\sigma_1$  and  $\sigma_2$  unknown, the test statistic for  $\mu_1 - \mu_2$  is:

### Test Statistic for Mean Difference (Independent Samples)

$$t^* = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_{0,1} - \mu_{0,2})}{\sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)}}$$

Next larger number than

$$df = \left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2 \bigg/ \left(\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}\right)$$

where  $df$  is either calculated or smaller of  $df_1$ , or  $df_2$  (10.9)

Actually, this is for  $\sigma_1 \neq \sigma_2$ .

If using a computer program.

If not using a computer program.

Go through the same five hypothesis testing steps.

Need normal populations to use  $t$  critical values.

# 9: Inferences Involving One Population

## 9.2 Inference about the Binomial Probability of Success

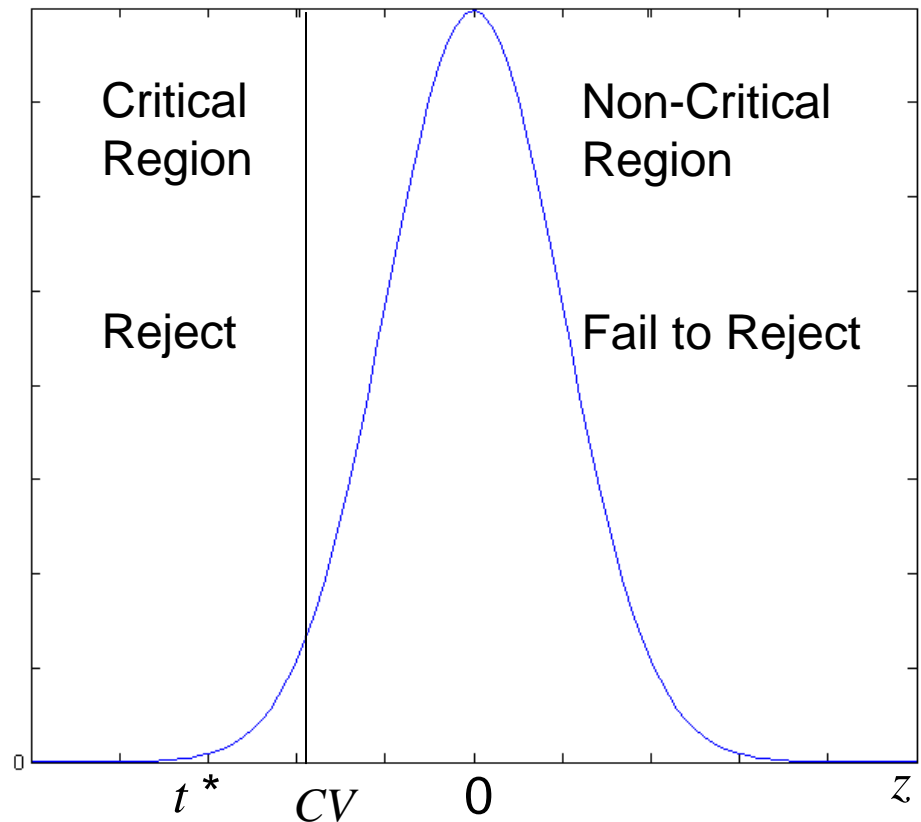
There are three possible hypothesis pairs for the difference in means.

$$H_0: \mu_1 - \mu_2 \geq 0 \text{ vs. } H_a: \mu_1 - \mu_2 < 0$$

Reject  $H_0$  if            less than

$$t^* = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}} \quad -t(df, \alpha)$$

data indicates  $\mu_1 - \mu_2 < 0$   
because  $\bar{x}_1 - \bar{x}_2$  is “a lot”  
smaller than 0.



# 9: Inferences Involving One Population

## 9.2 Inference about the Binomial Probability of Success

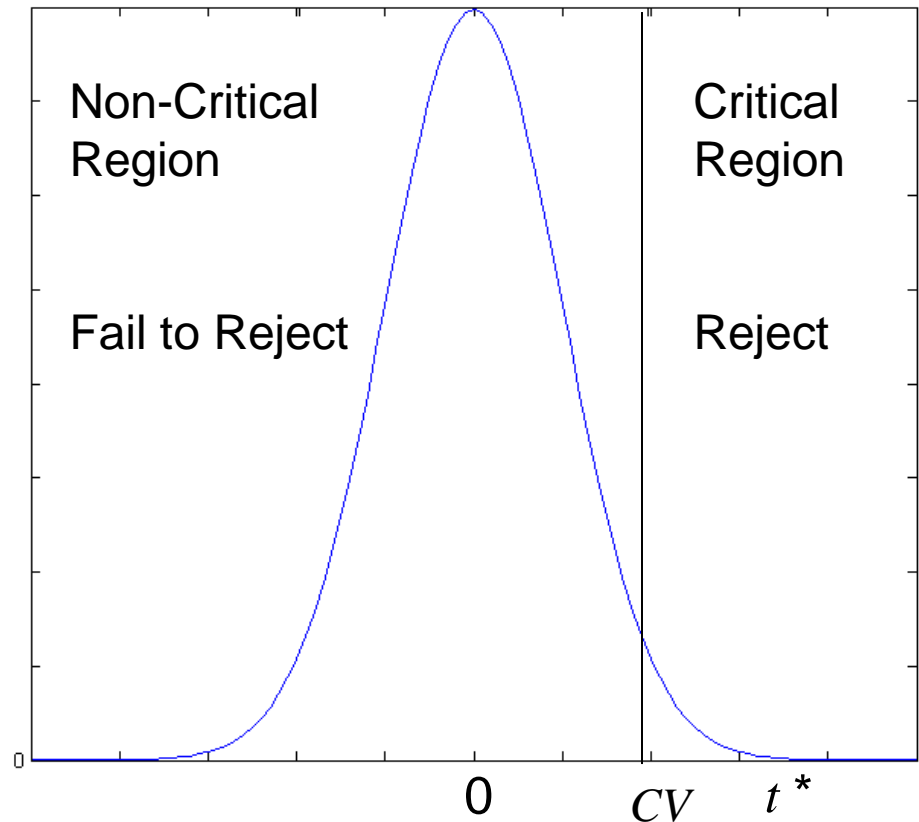
There are three possible hypothesis pairs for the difference in means.

$$H_0: \mu_1 - \mu_2 \leq 0 \text{ vs. } H_a: \mu_1 - \mu_2 > 0$$

Reject  $H_0$  if  $t$  is greater than

$$t^* = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}} \quad t(df, \alpha)$$

data indicates  $\mu_1 - \mu_2 > 0$   
because  $\bar{x}_1 - \bar{x}_2$  is “a lot”  
smaller than 0.



# 9: Inferences Involving One Population

## 9.2 Inference about the Binomial Probability of Success

There are three possible hypothesis pairs for the difference in means.

$$H_0: \mu_1 - \mu_2 = 0 \text{ vs. } H_a: \mu_1 - \mu_2 \neq 0$$

Reject  $H_0$  if less than

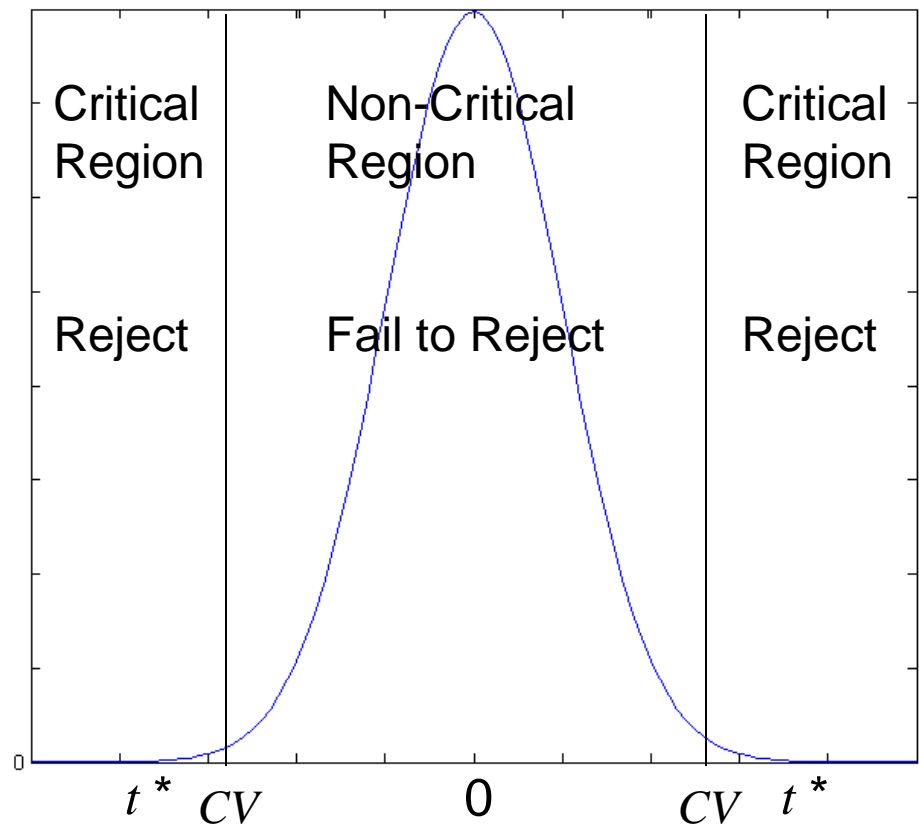
$$t^* = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}} \quad -t(df, \alpha / 2)$$

or if is greater than

$$t^* = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}} \quad t(df, \alpha / 2)$$

data indicates  $\mu_1 - \mu_2 \neq 0$ ,  $\bar{x}_1 - \bar{x}_2$

far from 0.

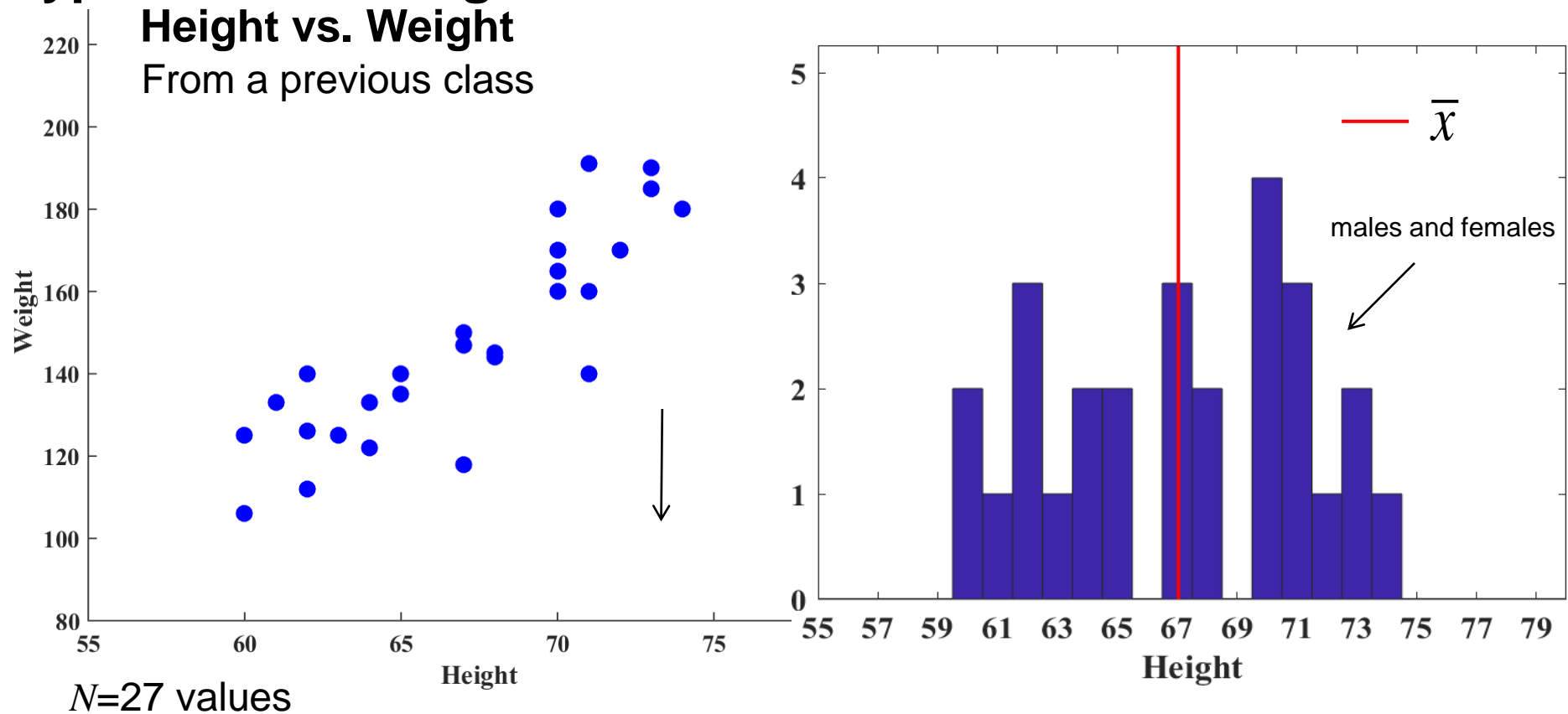


# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples

### Hypothesis Testing Procedure

27 values

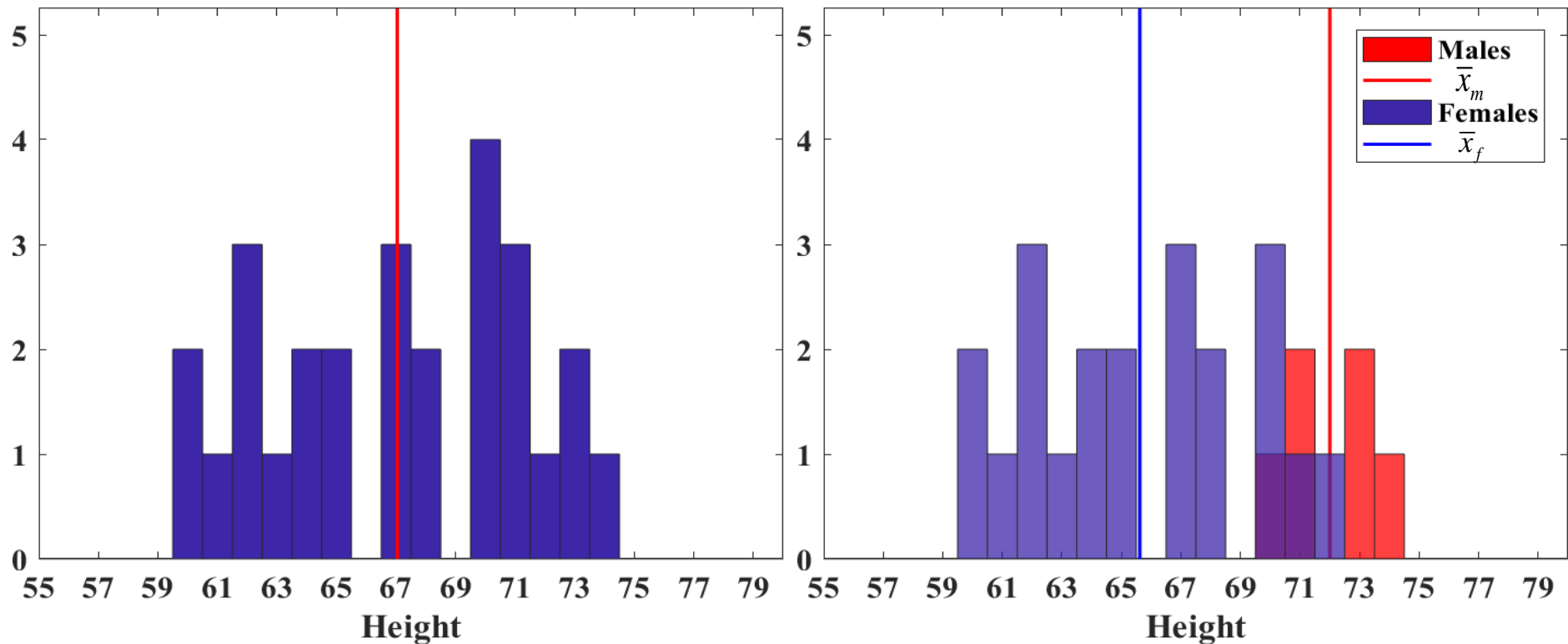


# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples

### Hypothesis Testing Procedure

27 values



Is the height of males = height of females at  $\alpha=.05$ ?

# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples

### Hypothesis Testing Procedure

27 values

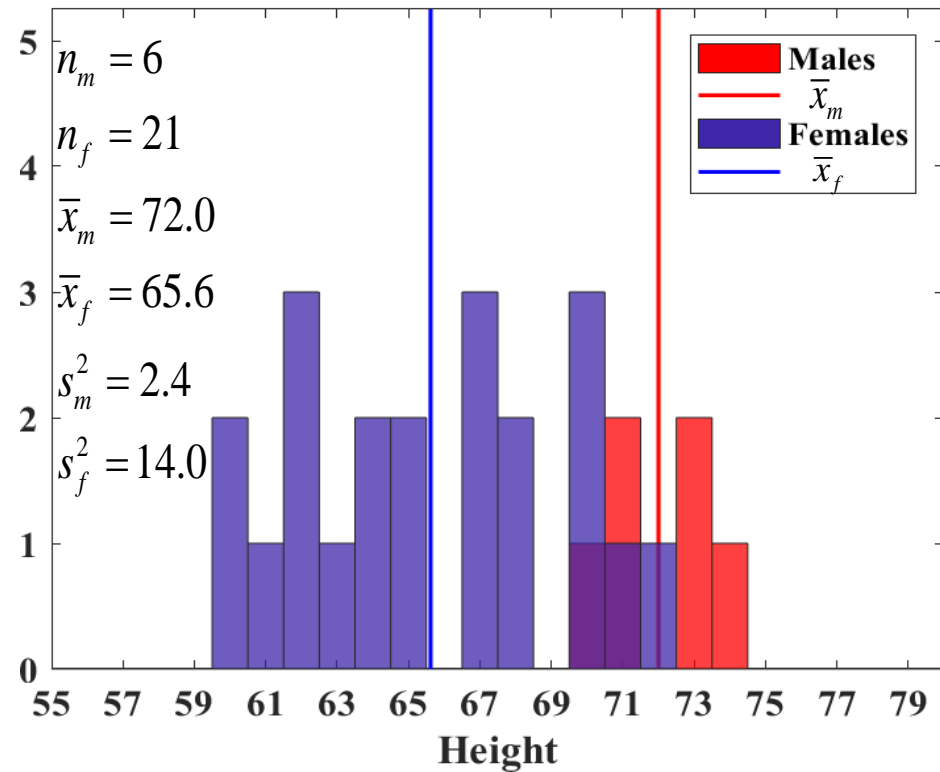
Step 1

Step 2

Step 3

Step 4

Step 5



# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples

### Hypothesis Testing Procedure

27 values

#### Step 1

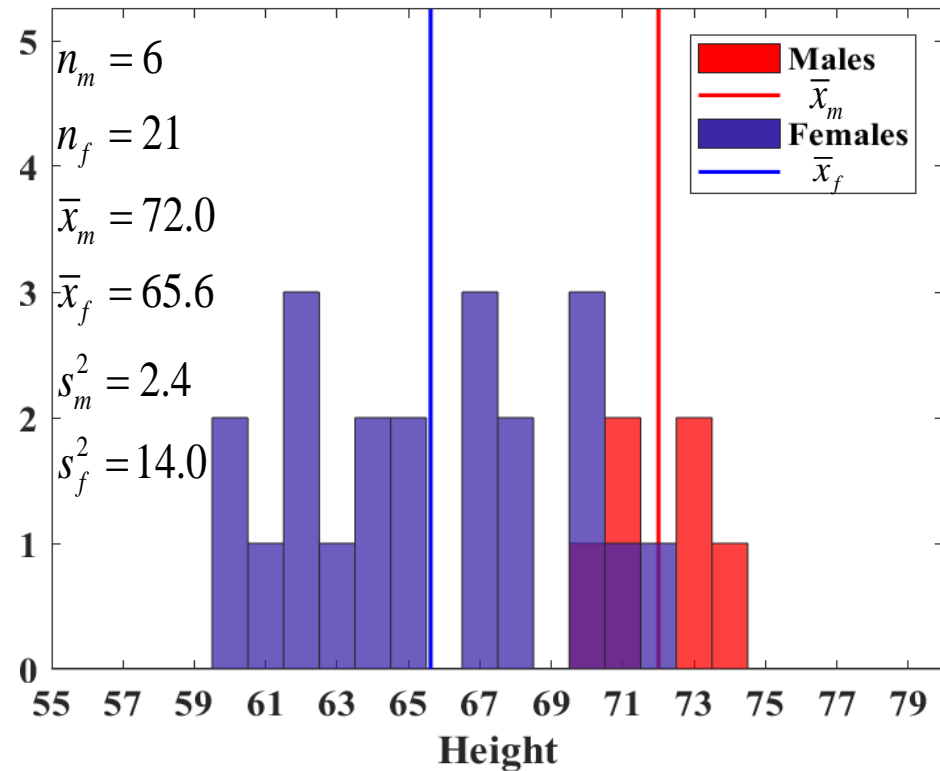
$$H_0: \mu_f = \mu_m \text{ vs. } H_a: \mu_f \neq \mu_m$$

#### Step 2

#### Step 3

#### Step 4

#### Step 5





# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples

### Hypothesis Testing Procedure

27 values

Step 1

$$H_0: \mu_f = \mu_m \text{ vs. } H_a: \mu_f \neq \mu_m$$

Step 2

$$t^* = \frac{(\bar{x}_m - \bar{x}_f) - (\mu_m - \mu_f)}{\sqrt{\left(\frac{s_m^2}{n_m}\right) + \left(\frac{s_f^2}{n_f}\right)}}$$

$$df = 5$$

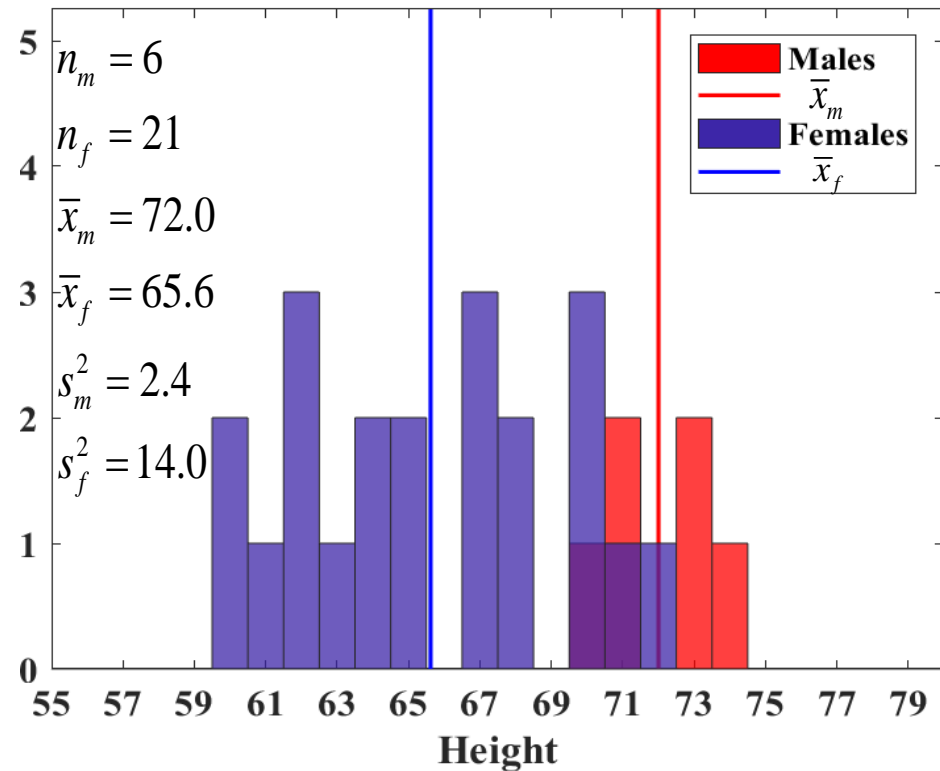
$$\alpha = .05$$

$$\sqrt{\left(\frac{s_m^2}{n_m}\right) + \left(\frac{s_f^2}{n_f}\right)}$$

Step 3

Step 4

Step 5



# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples

### Hypothesis Testing Procedure

27 values

#### Step 1

$$H_0: \mu_f = \mu_m \text{ vs. } H_a: \mu_f \neq \mu_m$$

#### Step 2

$$t^* = \frac{(\bar{x}_m - \bar{x}_f) - (\mu_m - \mu_f)}{\sqrt{\left(\frac{s_m^2}{n_m}\right) + \left(\frac{s_f^2}{n_f}\right)}}$$

$$df = 5$$

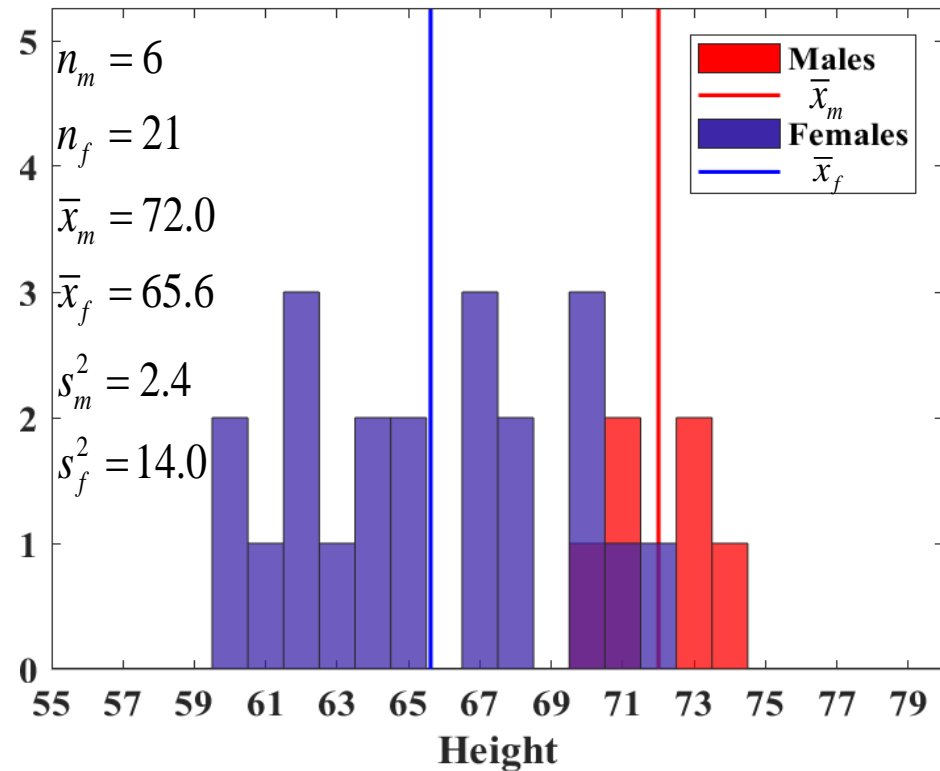
$$\alpha = .05$$

#### Step 3

$$t^* = \frac{(72.0 - 65.6) - (0)}{\sqrt{\left(\frac{2.4}{6}\right) + \left(\frac{14.0}{21}\right)}} = 6.17$$

#### Step 4

#### Step 5



# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples

### Hypothesis Testing Procedure

27 values

#### Step 1

$$H_0: \mu_f = \mu_m \text{ vs. } H_a: \mu_f \neq \mu_m$$

#### Step 2

$$t^* = \frac{(\bar{x}_m - \bar{x}_f) - (\mu_m - \mu_f)}{\sqrt{\left(\frac{s_m^2}{n_m}\right) + \left(\frac{s_f^2}{n_f}\right)}}$$

$$df = 5$$

$$\alpha = .05$$

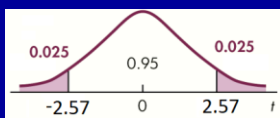
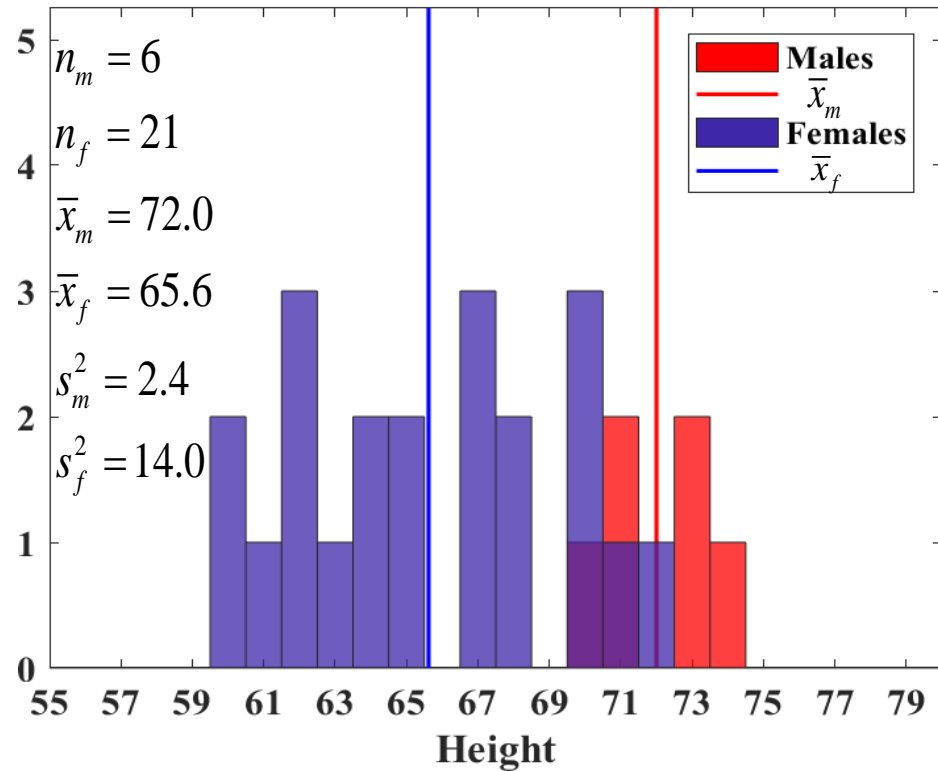
#### Step 3

$$t^* = \frac{(72.0 - 65.6) - (0)}{\sqrt{\left(\frac{2.4}{6}\right) + \left(\frac{14.0}{21}\right)}} = 6.17$$

#### Classical Step 4

$$t(df, \alpha / 2) = 2.57$$

#### Classical Step 5



# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples

### Hypothesis Testing Procedure

27 values

**Step 1**

$$H_0: \mu_f = \mu_m \text{ vs. } H_a: \mu_f \neq \mu_m$$

**Step 2**

$$t^* = \frac{(\bar{x}_m - \bar{x}_f) - (\mu_m - \mu_f)}{\sqrt{\left(\frac{s_m^2}{n_m}\right) + \left(\frac{s_f^2}{n_f}\right)}}$$

$$df = 5$$

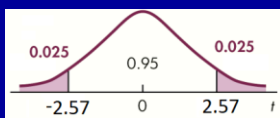
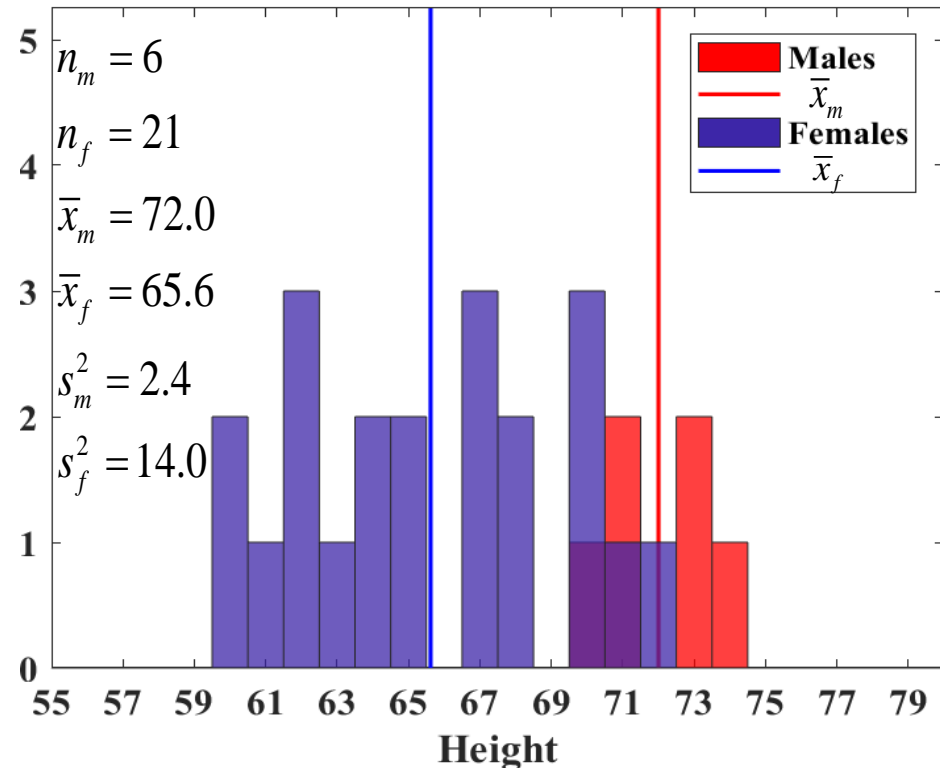
$$\alpha = .05$$

**Step 3**

$$t^* = \frac{(72.0 - 65.6) - (0)}{\sqrt{\left(\frac{2.4}{6}\right) + \left(\frac{14.0}{21}\right)}} = 6.17$$

Classical **Step 4**

$t(df, \alpha / 2) = 2.57$  **Step 5** Classical **Step 5** Reject  $H_0$   $6.17 > 2.57$ , height males  $\neq$  height females



# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples

### Hypothesis Testing Procedure

27 values

#### Step 1

$$H_0: \mu_f = \mu_m \text{ vs. } H_a: \mu_f \neq \mu_m$$

$$\text{Step 2 } t^* = \frac{(\bar{x}_m - \bar{x}_f) - (\mu_m - \mu_f)}{\sqrt{\left(\frac{s_m^2}{n_m}\right) + \left(\frac{s_f^2}{n_f}\right)}}$$

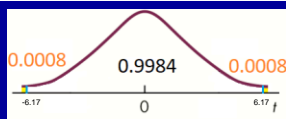
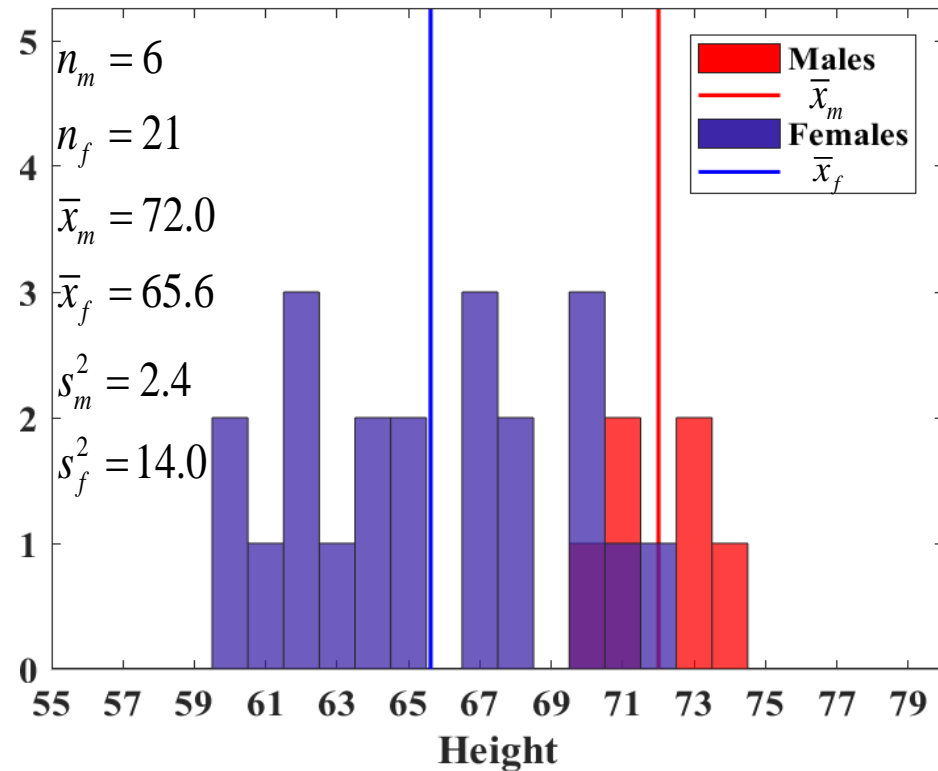
$$df = 5$$

$$\alpha = .05$$

$$\text{Step 3 } t^* = \frac{(72.0 - 65.6) - (0)}{\sqrt{\left(\frac{2.4}{6}\right) + \left(\frac{14.0}{21}\right)}} = 6.17$$

#### Step 4

$$P(|t^*| > 6.17) = .0016 \quad \text{Step 5}$$



# 10: Inferences Involving Two Populations

## 10.3 Inference for Mean Difference Two Independent Samples

### Hypothesis Testing Procedure

27 values

**Step 1**

$$H_0: \mu_f = \mu_m \text{ vs. } H_a: \mu_f \neq \mu_m$$

**Step 2**

$$t^* = \frac{(\bar{x}_m - \bar{x}_f) - (\mu_m - \mu_f)}{\sqrt{\left(\frac{s_m^2}{n_m}\right) + \left(\frac{s_f^2}{n_f}\right)}}$$

$$df = 5$$

$$\alpha = .05$$

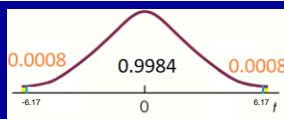
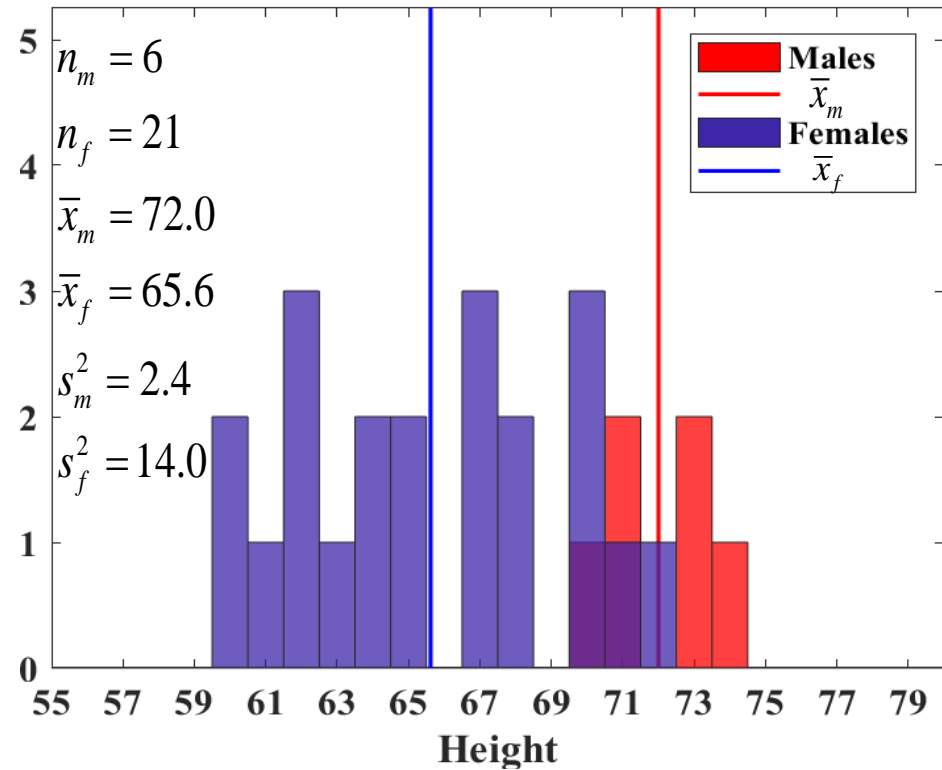
**Step 3**

$$t^* = \frac{(72.0 - 65.6) - (0)}{\sqrt{\left(\frac{2.4}{6}\right) + \left(\frac{14.0}{21}\right)}} = 6.17$$

**Step 4**

$$p\text{-value}$$

$P(|t^*| > 6.17) = .0016$  **Step 5**  $p\text{-value}$   $.002 < .05$ , height males  $\neq$  height females



# Chapter 10: Inferences Involving Two Populations

Questions?

Homework: Read Chapter 10.3

WebAssign

Chapter 10 # 41, 45, 53, 57, 58, 59, 63