

# Class 13

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# Agenda:

**Recap Chapter 8.1 - 8.2**

**Lecture Chapter 8.3 - 8.5**

# Recap Chapter 8.1-8.2

# 8: Introduction to Statistical Inference

## 8.1 The Nature of Estimation

**Point estimate for a parameter:** A single number ..., to estimate a parameter ... usually the .. **sample statistic.**

i.e.  $\bar{x}$  is a point estimate for  $\mu$

**Interval estimate:** An interval bounded by two values and used to estimate the value of a population parameter. ....

i.e.  $\bar{x} \pm$  (some amount) is an interval estimate for  $\mu$ .

point estimate  $\pm$  some amount

# 8: Introduction to Statistical Inference

## 8.1 The Nature of Estimation

**Significance Level:** Probability parameter outside interval,  $\alpha$ .

$$P(\mu \text{ not in } \bar{x} \pm \text{some amount}) = \alpha$$

**Level of Confidence  $1-\alpha$ :**

$$P(\bar{x} - \text{some amount} < \mu < \bar{x} + \text{some amount}) = 1 - \alpha$$

**Confidence Interval:**

point estimator  $\pm$  some amount that depends on

confidence level

# 8: Introduction to Statistical Inference

## 8.2 Estimation of Mean $\mu$ ( $\sigma$ Known)

By SDSM

$$\mu_{\bar{x}} = \mu,$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

What this implies is that  $z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$

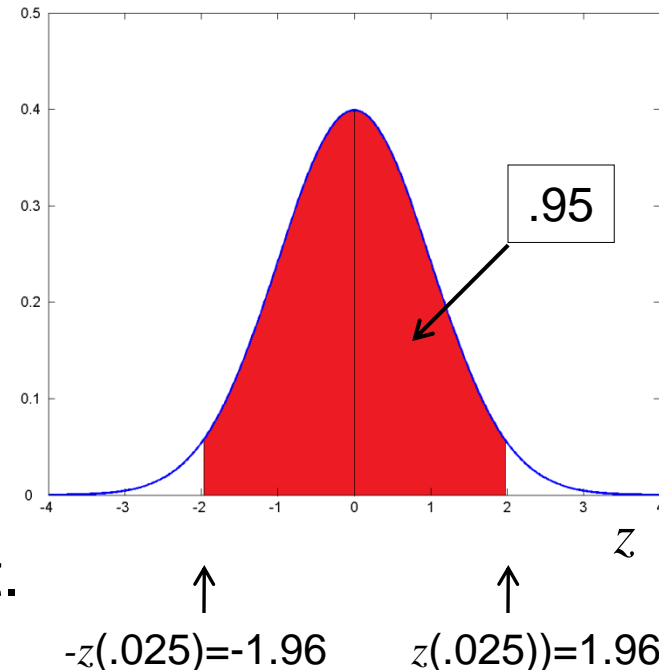
has an approximate standard normal distribution!

$$P(-1.96 < z < 1.96) = 0.95 \quad \alpha = .05$$

Or more generally,

$$P(-z(\alpha / 2) < z < z(\alpha / 2)) = 1 - \alpha$$

$z(\alpha / 2)$  called the confidence coefficient.



## 8: Introduction to Statistical Inference

### 8.2 Estimation of Mean $\mu$ ( $\sigma$ Known)

Thus, a  $(1-\alpha)\times 100\%$  confidence interval for  $\mu$  is

$$\bar{x} \pm z(\alpha / 2) \frac{\sigma}{\sqrt{n}}$$

which if  $\alpha=0.05$ , a 95% confidence interval for  $\mu$  is

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}} \quad z(.025)=1.96$$

#### Confidence Interval for Mean:

$$\bar{x} - z(\alpha / 2) \frac{\sigma}{\sqrt{n}} \quad \text{to} \quad \bar{x} + z(\alpha / 2) \frac{\sigma}{\sqrt{n}} \quad (8.1)$$

# Chapter 8: Introduction to Statistical Inference

Questions?

Homework: Read Chapter 8.1-8.2

WebAssign

Chapter 8 # 5, 15, 22, 24, 35, 47



# Lecture Chapter 8.3 - 8.5

# 8: Introduction to Statistical Inference

## 8.3 The Nature of Hypothesis Testing

We make decisions every day in our lives.

Should I do  $A$  or should I do  $B$  (not  $A$ )?

**Hypothesis:** A statement that something is true.

Example:

**Proposed Hypothesis:**

“The party will be a great time.”

**Opposing Hypothesis:**

“The party will be a dud.”

# 8: Introduction to Statistical Inference

## 8.3 The Nature of Hypothesis Testing

How do you decide whether to go?

In order to determine which hypothesis we believe is true, we perform something called a **statistical hypothesis test**.

**Statistical hypothesis test:** A process by which a decision is made between two opposing hypotheses. ... (read the rest)

## 8: Introduction to Statistical Inference

### 8.3 The Nature of Hypothesis Testing

We call the proposed hypothesis the **null hypothesis** and the opposing hypothesis the **alternative hypothesis**.

**Null Hypothesis,  $H_0$ :** The hypothesis that we will test. Generally a statement that a parameter has a specific value.

**Alternative Hypothesis,  $H_a$ :** A statement about the same parameter that is used in the null hypothesis. ... parameter has a value different ... from the value in the null hypothesis.

# 8: Introduction to Statistical Inference

## 8.3 The Nature of Hypothesis Testing

**Example 1:** Friend's Party.

$H_0$ : "The party will be a dud"

vs.

$H_a$ : "The party will be a great time"

# 8: Introduction to Statistical Inference

## 8.3 The Nature of Hypothesis Testing

**Example 1:** Friend's Party.

$H_0$ : "The party will be a dud"

vs.

$H_a$ : "The party will be a great time"

**Example 2:** Math 1700 Students Height

$H_0$ : The mean height of Math 1700 students is 69",  $\mu = 69$ ".

vs.

$H_a$ : The mean height of Math 1700 students is not 69",  $\mu \neq 69$ ".

# 8: Introduction to Statistical Inference

## 8.3 The Nature of Hypothesis Testing

### Example 1: Friend's Party

$H_0$ : "The party will be a dud"

vs.

$H_a$ : "The party will be a great time"

Four outcomes from a hypothesis test.

	Party Great	Party a dud.
We go.	Correct Decision	Error
We do not go.	Error	Correct Decision

There is always a chance we make an error.

# 8: Introduction to Statistical Inference

## 8.3 The Nature of Hypothesis Testing

### Example 1: Friend's Party

$H_0$ : "The party will be a dud"

vs.

$H_a$ : "The party will be a great time"

If do not go to party and it's great,  
we made an error in judgment.

If go to party and it's a dud,  
we made in error in judgment.

Four outcomes from a hypothesis test.

	Party Great	Party a dud.
We go.	Correct Decision	Error
We do not go.	Error	Correct Decision

There is always a chance  
we make an error.



# 8: Introduction to Statistical Inference

## 8.3 The Nature of Hypothesis Testing

### Example 2: Math 1700 Height

$$H_0: \mu = 69''$$

vs.

$$H_a: \mu \neq 69''$$

Four outcomes from a hypothesis test.

	$\mu = 69$	$\mu \neq 69$
Fail to reject $H_0$ .	Correct Decision	Error
Reject $H_0$ .	Error	Correct Decision

There is always a chance we make an error.

# 8: Introduction to Statistical Inference

## 8.3 The Nature of Hypothesis Testing

**Example 2:** Math 1700 Height

$$H_0: \mu = 69''$$

vs.

$$H_a: \mu \neq 69''$$

If we reject  $H_0$  and it is true, we made in error in judgment.

If we do not reject  $H_0$  and it is false, we have made an error in judgment.

Four outcomes from a hypothesis test.

	$\mu = 69$	$\mu \neq 69$
Fail to reject $H_0$ .	Correct Decision	Error
Reject $H_0$ .	Error	Correct Decision

There is always a chance we make an error.

# 8: Introduction to Statistical Inference

## 8.3 The Nature of Hypothesis Testing

**Type I Error:** When a true null hypothesis  $H_0$  is rejected.

**Type II Error:** When we decide in favor of a null hypothesis that is actually false.

Four outcomes from a hypothesis test.

	$H_0$ True	$H_0$ False
Fail to Reject $H_0$	Type A Correct Decision	Type II Error
Reject $H_0$	Type I Error	Type B Correct Decision

# 8: Introduction to Statistical Inference

## 8.3 The Nature of Hypothesis Testing

**Level of Significance ( $\alpha$ ):** The probability of committing a type I error. (Sometimes  $\alpha$  is called the false positive rate.)

	$H_0$ True	$H_0$ False
Fail to Reject $H_0$	Type A Correct Decision ( $1-\alpha$ )	Type II Error ( $\beta$ )
Reject $H_0$	Type I Error ( $\alpha$ )	Type B Correct Decision ( $1-\beta$ )

# 8: Introduction to Statistical Inference

## 8.3 The Nature of Hypothesis Testing

**Level of Significance ( $\alpha$ ):** The probability of committing a type I error. (Sometimes  $\alpha$  is called the false positive rate.)

**Type II Probability ( $\beta$ ):**  
The probability of committing a type II error.  
(Sometimes  $\beta$  is called the false negative rate.)

	$H_0$ True	$H_0$ False
Fail to Reject $H_0$	Type A Correct Decision ( $1-\alpha$ )	Type II Error ( $\beta$ )
Reject $H_0$	Type I Error ( $\alpha$ )	Type B Correct Decision ( $1-\beta$ )

## 8: Introduction to Statistical Inference

### 8.3 The Nature of Hypothesis Testing

We need to determine a measure that will quantify what we should believe.

**Test Statistic:** A random variable whose value is calculated from the sample data and is used in making the decision “reject  $H_0$ ” or “fail to reject  $H_0$ .”

# 8: Introduction to Statistical Inference

## 8.3 The Nature of Hypothesis Testing

We need to determine a measure that will quantify what we should believe.

**Test Statistic:** A random variable whose value is calculated from the sample data and is used in making the decision “reject  $H_0$ ” or “fail to reject  $H_0$ .”

### **Example 1:** Friend’s Party

Fraction of friends parties that were good.

### **Example 2:** Math 1700 Heights

Sample mean height.

## 8: Introduction to Statistical Inference

### 8.4 Hypothesis Test of Mean ( $\sigma$ Known): A Probability-Value Approach

Note the first item presented in this section!

**The assumption for hypothesis tests about mean  $\mu$  using known  $\sigma$ :** The sampling distribution of  $\bar{x}$  has a normal distribution.

Use normal distribution for  $\bar{x}$  to know when we have an unusual one compared to what we think it is.



# 8: Introduction to Statistical Inference

## 8.4 Hypothesis Test of Mean ( $\sigma$ Known): Probability Approach

### THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

Step 1 The Set-Up:

Step 2 The Hypothesis Test Criteria:

Step 3 The Sample Evidence:

Step 4 The Probability Distribution:

Step 5 The Results:

## 8: Introduction to Statistical Inference

### 8.4 Hypothesis Test of Mean ( $\sigma$ Known): Probability Approach

#### THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

##### Step 1 The Set-Up:

- a. Describe the population parameter of interest.

The population parameter of interest is the mean  $\mu$ , the height of Math 1700 students.

# 8: Introduction to Statistical Inference

## 8.4 Hypothesis Test of Mean ( $\sigma$ Known): Probability Approach

### THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

#### Step 1 The Set-Up:

- b. State the null hypothesis ( $H_0$ ) and the alternative hypotheses ( $H_a$ ).

#### Null Hypothesis

1. Greater than or equal to ( $\geq$ )
2. Less than or equal to ( $\leq$ )
3. Equal to ( $=$ )

#### Alternative Hypothesis

- Less than ( $<$ )  
 Greater than ( $>$ )  
 Not equal to ( $\neq$ )

$H_0: \mu = 69''$  vs.  $H_a: \mu \neq 69''$

Figure from Johnson & Kuby, 2012.

## 8: Introduction to Statistical Inference

### 8.4 Hypothesis Test of Mean ( $\sigma$ Known): Probability Approach

#### THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

##### Step 2 The Hypothesis Test Criteria:

- a. Check the assumptions.

Assume we know  $\sigma$  from past experience.

Assume that  $n$  is “large” so that by the CLT  $\bar{x}$  is normally distributed.

$$\mu_{\bar{x}} = \mu_0, \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Hypothesized value.

## 8: Introduction to Statistical Inference

### 8.4 Hypothesis Test of Mean ( $\sigma$ Known): Probability Approach

#### THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

##### Step 2 The Hypothesis Test Criteria:

- b. Identify the probability distribution and the test statistic to be used.

The standard normal distribution is to be used because  $\bar{x}$  is expected to have a normal distribution.

##### Test Statistic for Mean:

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad \text{where } \sigma \text{ is assumed to be known} \quad (8.4)$$

← Hypothesized value.

## 8: Introduction to Statistical Inference

### 8.4 Hypothesis Test of Mean ( $\sigma$ Known): Probability Approach

#### THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

##### Step 3 The Sample Evidence:

a. Collect a sample of information.

Take a random sample from a population with mean  $\mu$  that being questioned.

b. Calculate the value of the test statistic.

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{67.2 - 69}{4 / \sqrt{15}} = -1.74$$

Assuming  $n=15$  and 67.2 is sample mean. With known  $\sigma = 4$ .

## 8: Introduction to Statistical Inference

### 8.4 Hypothesis Test of Mean ( $\sigma$ Known): Probability Approach

#### THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

##### Step 4 The Probability Distribution:

- a. Calculate the  $p$ -value for the test statistic.

**Probability value, or  $p$ -value:** The probability that the test statistic could be the value it is or a more extreme value.

$$H_0: \mu = 69'' \text{ vs. } H_a: \mu \neq 69''$$

$$P(z > |z^*|) = p\text{-value}$$

since two sided test.

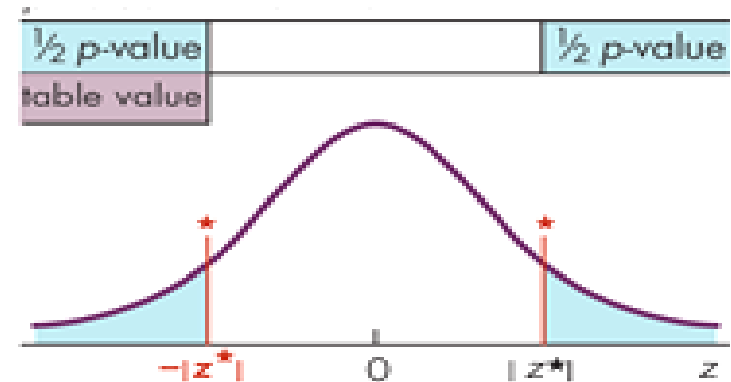


Figure from Johnson & Kuby, 2012.

## 8: Introduction to Statistical Inference

### 8.4 Hypothesis Test of Mean ( $\sigma$ Known): Probability Approach

#### THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

##### Step 5 The Results:

a. State the decision about  $H_0$ .

Is the  $p$ -value small enough to show that the sample evidence is highly unlikely if the null hypothesis were true?

##### Decision rule:

a. If the  $p$ -value is *less than or equal to* the level of significance  $\alpha$ , then the decision must be **reject  $H_0$** .

b. If the  $p$ -value is *greater than* the level of significance  $\alpha$ , then the decision must be **fail to reject  $H_0$** .



## 8: Introduction to Statistical Inference

### 8.4 Hypothesis Test of Mean ( $\sigma$ Known): Probability Approach

#### THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

##### Step 5 The Results:

b. State the conclusion about  $H_a$ .

With  $\alpha = 0.05$ ,  $p\text{-value} = 0.0819$ .

Therefore there is not sufficient evidence to reject  $H_0$ .

Fail to reject  $H_0$ .

$$P(z < -1.74) = 0.0409$$

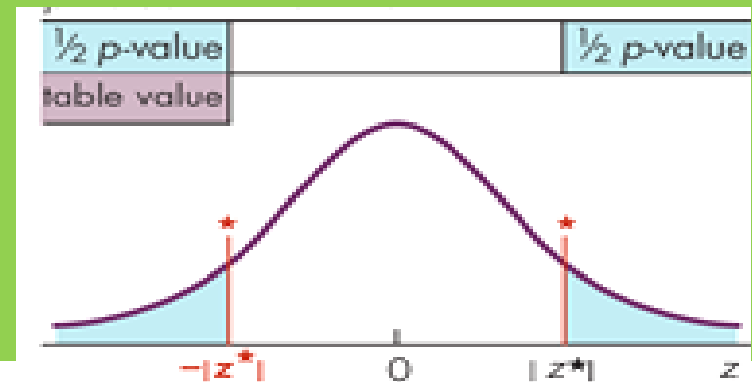


Figure from Johnson & Kubly, 2008.

# 8: Introduction to Statistical Inference

## 8.5 Hypothesis Test of Mean $\mu$ ( $\sigma$ Known):

### A Classical Approach

#### THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

Step 1 The Set-Up:

Step 2 The Hypothesis Test Criteria:

Step 3 The Sample Evidence:

Step 4 The Probability Distribution:

Step 5 The Results:

## 8: Introduction to Statistical Inference

### 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

#### THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

##### Step 1 The Set-Up:

- a. Describe the population parameter of interest.

The population parameter of interest is the mean  $\mu$ , the height of Math 1700 students.

# 8: Introduction to Statistical Inference

## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Probability Approach

### THE PROBABILITY-VALUE HYPOTHESIS TEST: 5 STEPS

#### Step 1 The Set-Up:

- b. State the null hypothesis ( $H_0$ ) and the alternative hypotheses ( $H_a$ ).

#### Null Hypothesis

1. Greater than or equal to ( $\geq$ )
2. Less than or equal to ( $\leq$ )
3. Equal to ( $=$ )

#### Alternative Hypothesis

- Less than ( $<$ )
- Greater than ( $>$ )
- Not equal to ( $\neq$ )

$H_0: \mu = 69''$  vs.  $H_a: \mu \neq 69''$

Figure from Johnson & Kuby, 2012.

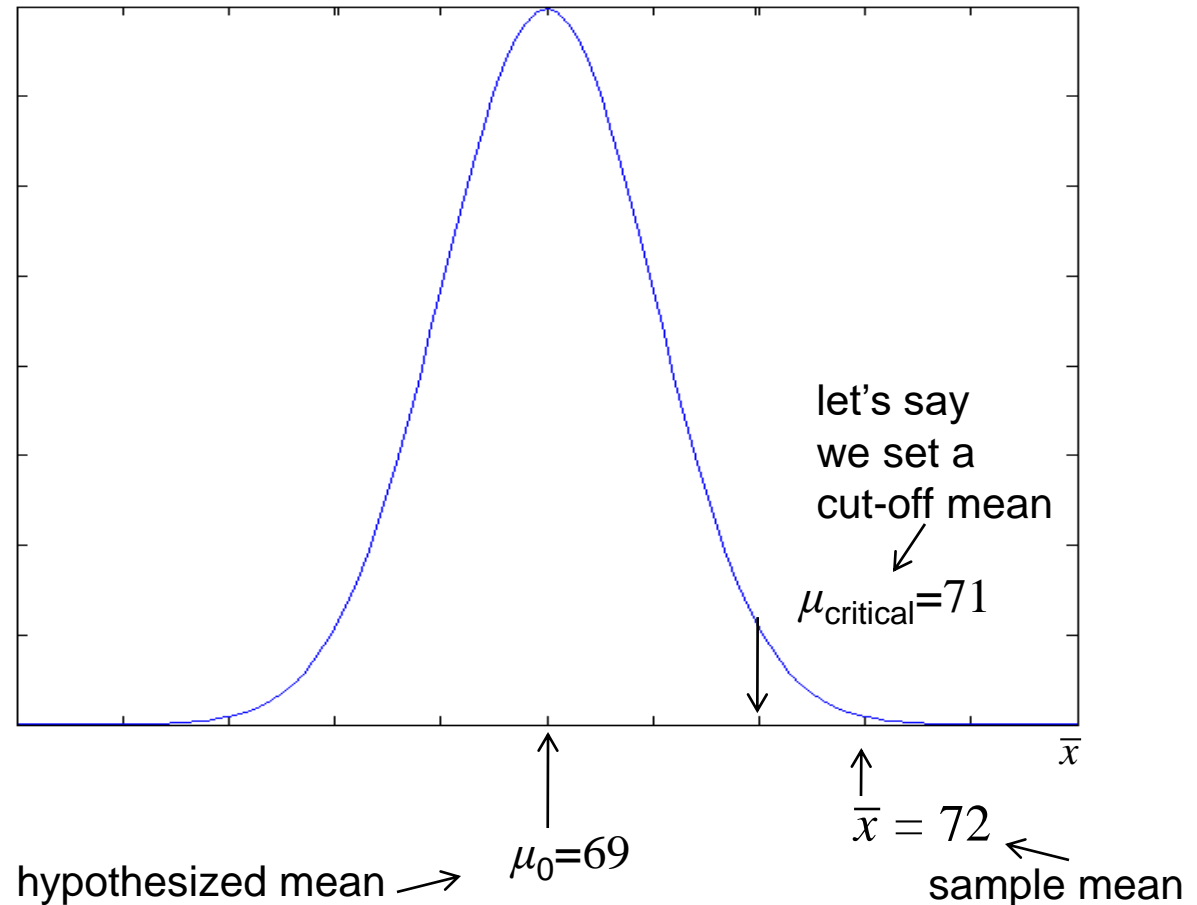
# 8: Introduction to Statistical Inference

## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

### Scenario:

True  $\mu$  not likely  
" $\mu_0=69$ ".

Reject  $H_0: \mu=\mu_0$   
(do not believe  $H_0$ )



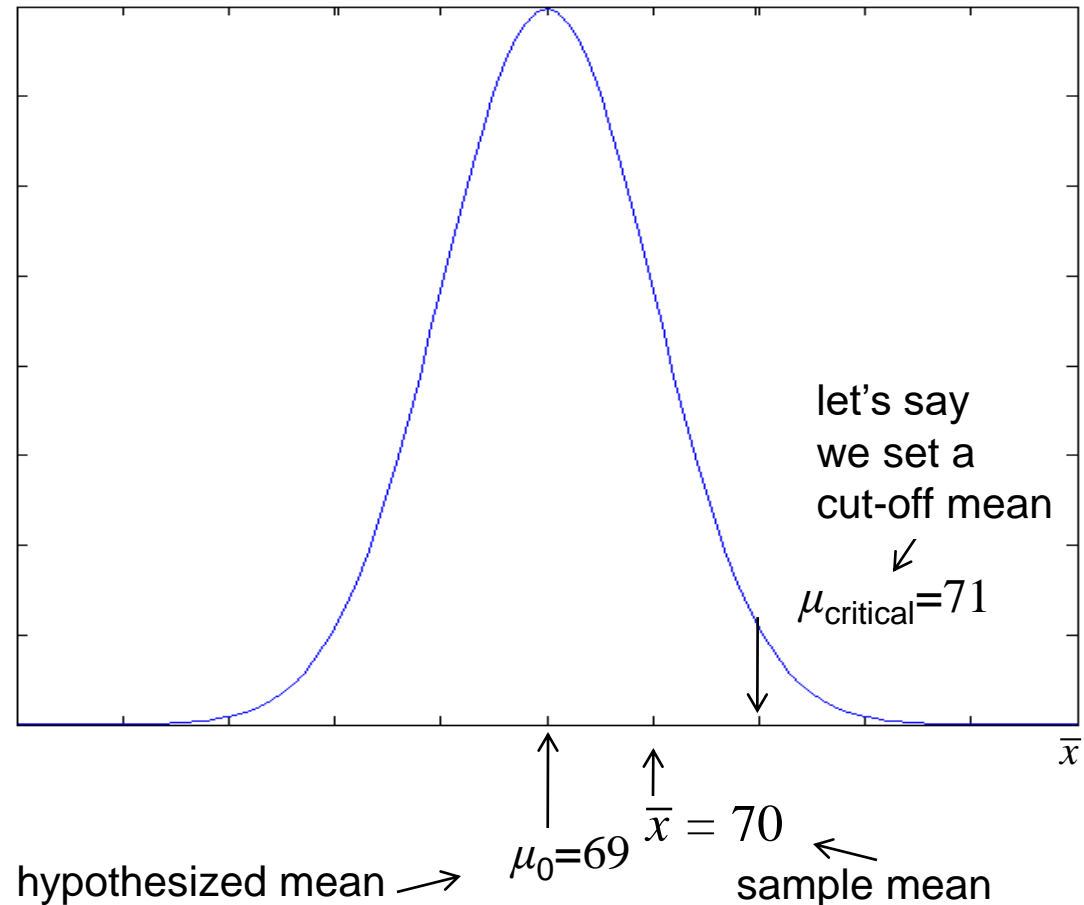
# 8: Introduction to Statistical Inference

## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

### Scenario:

True  $\mu$  likely  
 $\mu_0 = 69$ .

Fail to reject  $H_0: \mu = \mu_0$   
(not enough evidence  
not to believe  $H_0$ )



## 8: Introduction to Statistical Inference

### 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

We need a “better” (objective) way to set a “cut-off “ value or “cut-off” values for which we would either believe  $H_0$  or for which we would not have enough evidence not believe  $H_0$ .

We need to use the normal distribution and probabilities.

## 8: Introduction to Statistical Inference

### 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

#### THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

##### Step 2 The Hypothesis Test Criteria:

- a. Check the assumptions.

Assume we know from past experience that  $\sigma=4$ .

Assume that  $n$  is “large” so that by the CLT  $\bar{x}$  is normally distributed.

$$\mu_{\bar{x}} = \mu, \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad \text{by SDSM}$$



## 8: Introduction to Statistical Inference

### 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

#### THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

##### Step 2 The Hypothesis Test Criteria:

- b. Identify the probability distribution and the test statistic to be used.

The standard normal distribution is to be used because  $\bar{x}$  is expected to have a normal distribution.

##### Test Statistic for Mean:

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad \text{where } \sigma \text{ is assumed to be known} \quad (8.4)$$

## 8: Introduction to Statistical Inference

### 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

#### THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

##### Step 2 The Hypothesis Test Criteria:

- c. Determine the level of significance,  $\alpha$ .

After much consideration, we assign a tolerable probability of a Type I error to be  $\alpha=0.05$ .

**Type I Error:** When a true null hypothesis  $H_0$  is rejected.

## 8: Introduction to Statistical Inference

### 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

#### THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

##### Step 3 The Sample Evidence:

a. Collect a sample of information.

Take a random sample from the population with mean  $\mu$  that being questioned.

b. Calculate the value of the test statistic.

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{67.2 - 69}{4 / \sqrt{15}} = -1.74$$

Assuming  $n=15$  and 67.2 is sample mean. With known  $\sigma = 4$ .

## 8: Introduction to Statistical Inference

### 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

#### THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

##### Step 4 The Probability Distribution:

- a. Determine the critical region and critical value(s).

**Critical Region:** The set of values for the test statistic that will cause us to reject the null hypothesis.

**Critical value(s):** The “first” or “boundary” value(s) of the critical region(s).

# 8: Introduction to Statistical Inference

## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

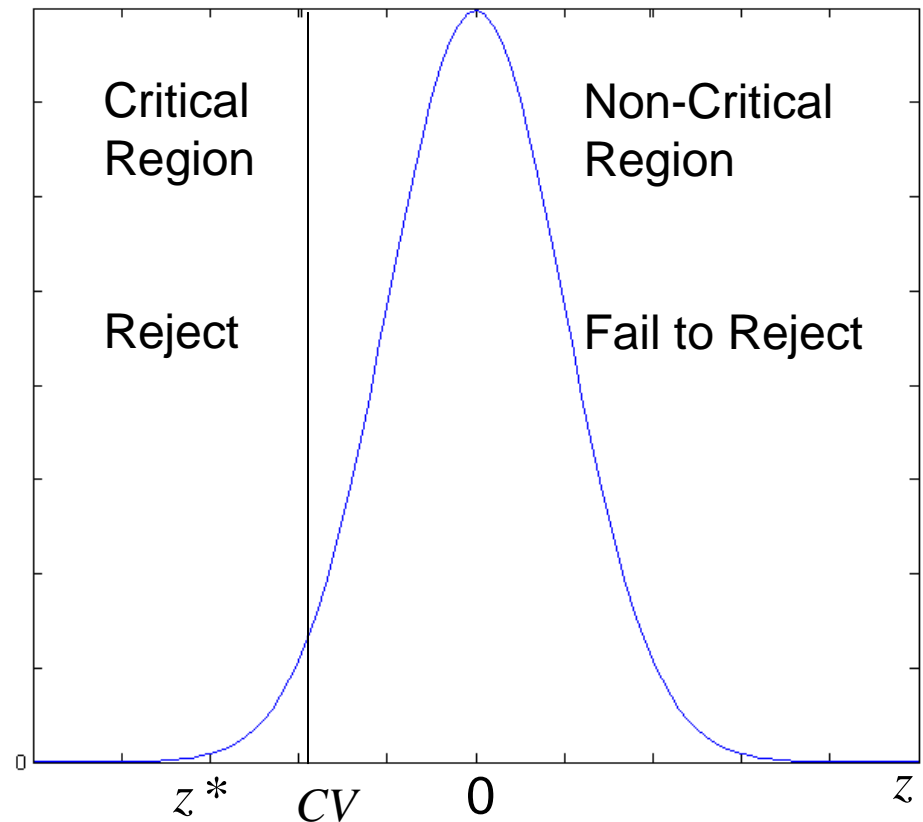
There are three possible hypothesis pairs for the mean.

$$H_0: \mu \geq \mu_0 \text{ vs. } H_a: \mu < \mu_0$$

Reject  $H_0$  if  $z^*$  is less than

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad -z(\alpha)$$

data indicates  $\mu < \mu_0$   
because  $\bar{x}$  is “a lot”  
smaller than  $\mu_0$



# 8: Introduction to Statistical Inference

## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

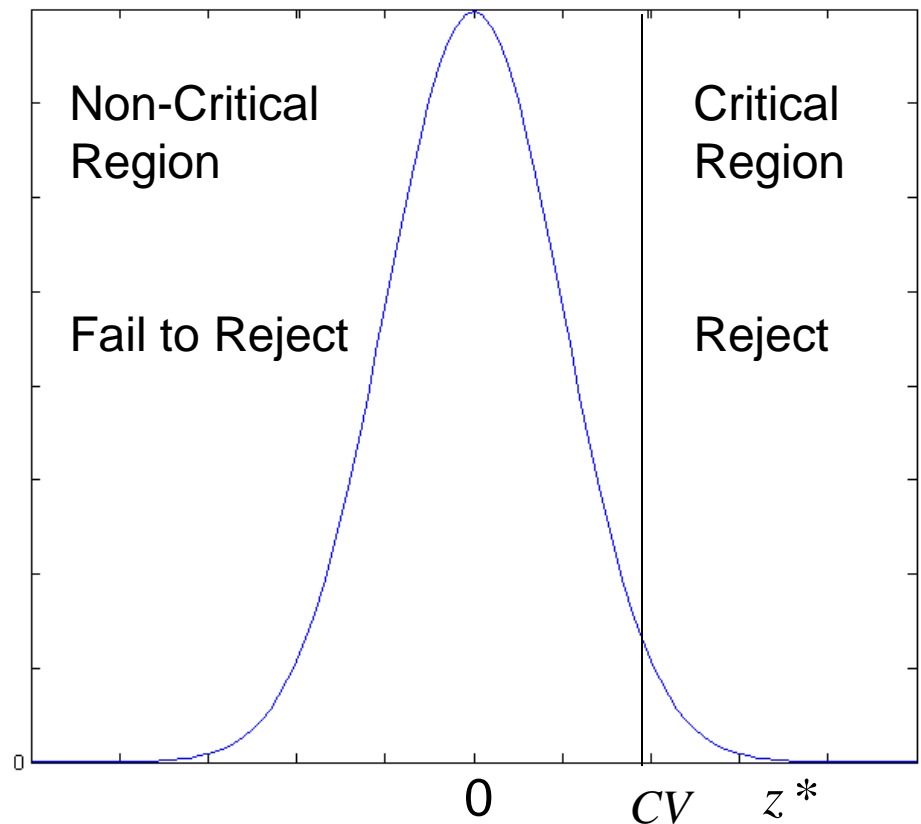
There are three possible hypothesis pairs for the mean.

$$H_0: \mu \leq \mu_0 \text{ vs. } H_a: \mu > \mu_0$$

Reject  $H_0$  if  $z$  greater than

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad z(\alpha)$$

data indicates  $\mu > \mu_0$   
because  $\bar{x}$  is “a lot”  
larger than  $\mu_0$



# 8: Introduction to Statistical Inference

## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

There are three possible hypothesis pairs for the mean.

$$H_0: \mu = \mu_0 \text{ vs. } H_a: \mu \neq \mu_0$$

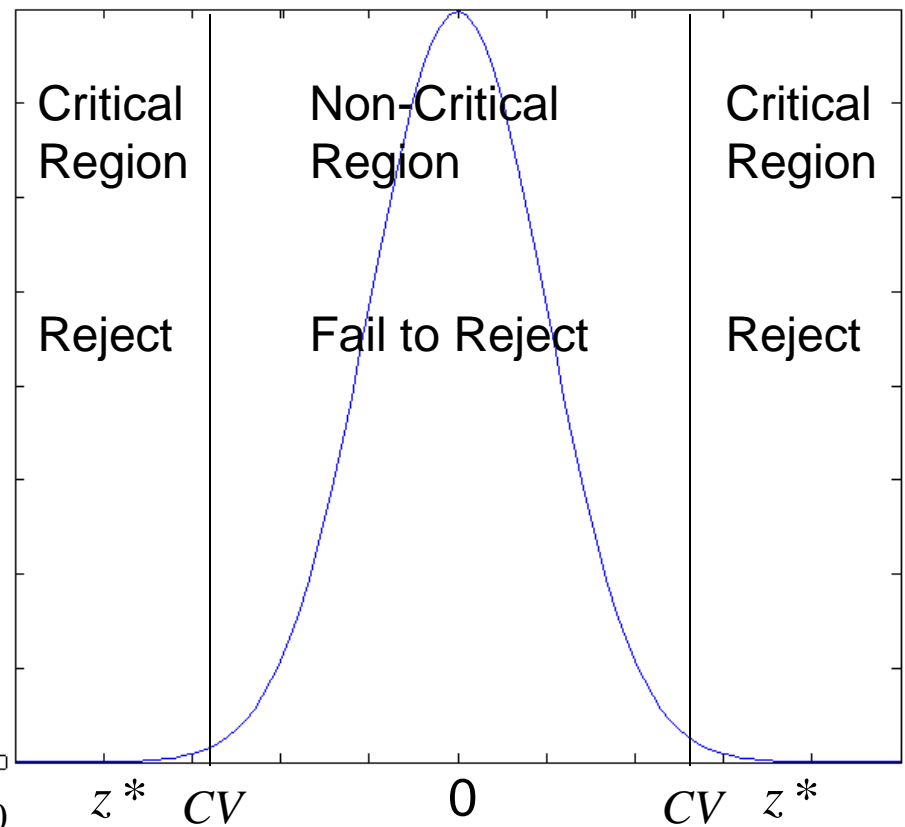
Reject  $H_0$  if less than

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} < -z(\alpha / 2)$$

or if greater than

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} > z(\alpha / 2)$$

data indicates  $\mu \neq \mu_0$ ,  $\bar{x}$  far from  $\mu_0$



# 8: Introduction to Statistical Inference

## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

### THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

#### Step 4 The Probability Distribution:

- Determine the critical region and critical value(s).
- Determine whether or not the calculated test statistic is in the critical region.

critical region	noncritical region	critical region
-----------------	--------------------	-----------------

$$H_0: \mu = 69'' \text{ vs. } H_a: \mu \neq 69''$$

$$P(z > z(\alpha / 2)) = \alpha / 2 \quad ,$$

$$z(.025) = 1.96$$

since two sided test.

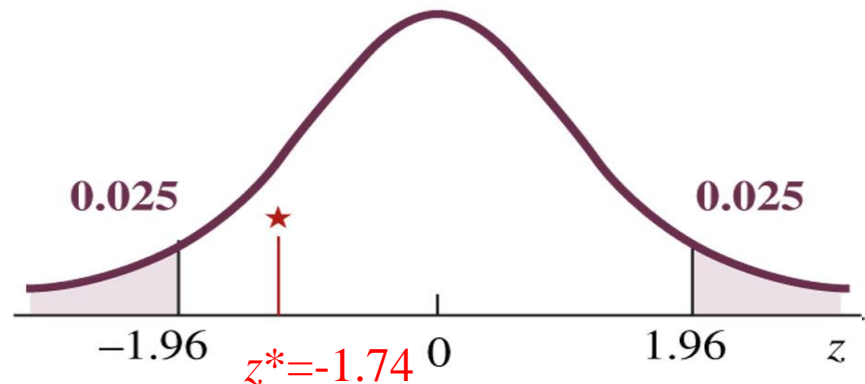


Figure from Johnson & Kubly, 2012.



## 8: Introduction to Statistical Inference

### 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

#### THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

##### Step 5 The Results:

- a. State the decision about  $H_0$ .  
Need a decision rule.

##### Decision rule:

- a. If the test statistic falls *within the critical region*, then the decision must be **reject  $H_0$** .
- b. If the test statistic is *not in the critical region*, then the decision must be **fail to reject  $H_0$** .

## 8: Introduction to Statistical Inference

### 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

#### THE CLASSICAL HYPOTHESIS TEST: 5 STEPS

##### Step 5 The Results:

b. State the conclusion about  $H_a$ .

With  $\alpha = 0.05$ ,

there is not sufficient evidence to reject  $H_0$ .

Fail to reject  $H_0$ .

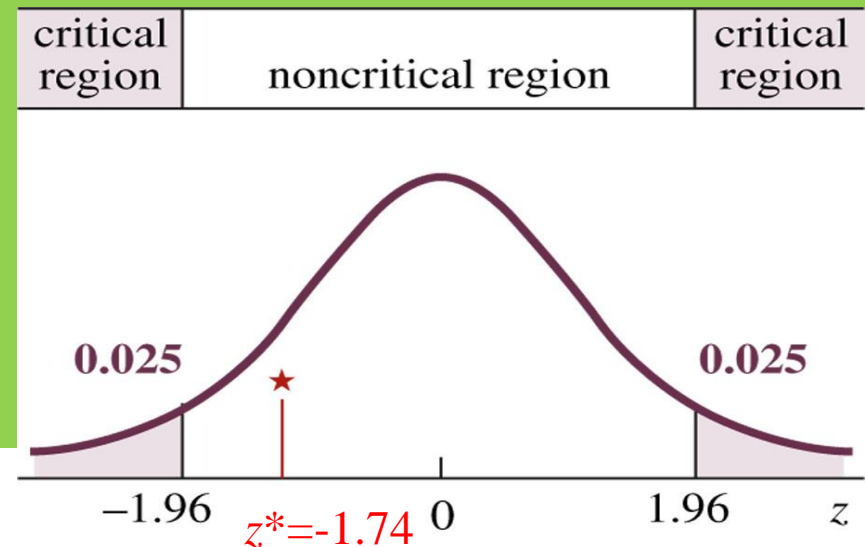


Figure from Johnson & Kubly, 2012.

## 8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

### 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

Let's examine the hypothesis test

$$H_0: \mu \leq 69'' \text{ vs. } H_a: \mu > 69''$$

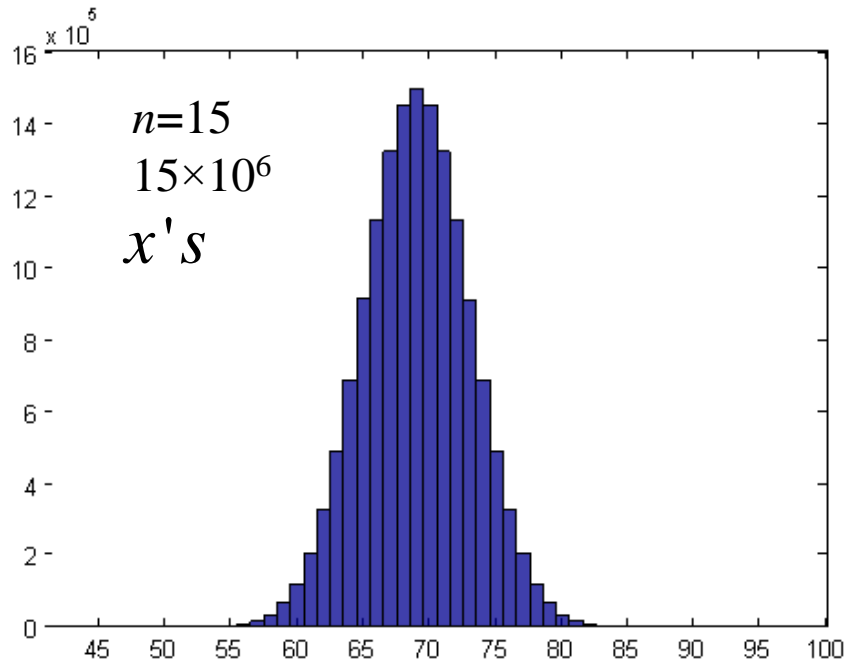
with  $\alpha=0.05$  for the heights of Math 1700 students.

Generate random data values.

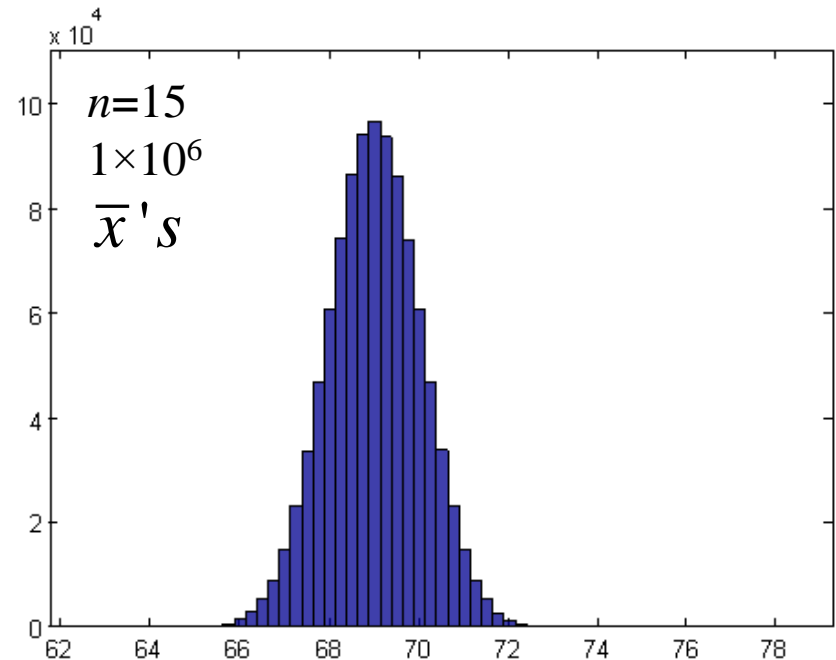
# 8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

Generated  $15 \times 10^6$   
normal data values  
from  $\mu = 69''$  and  $\sigma = 4''$ .

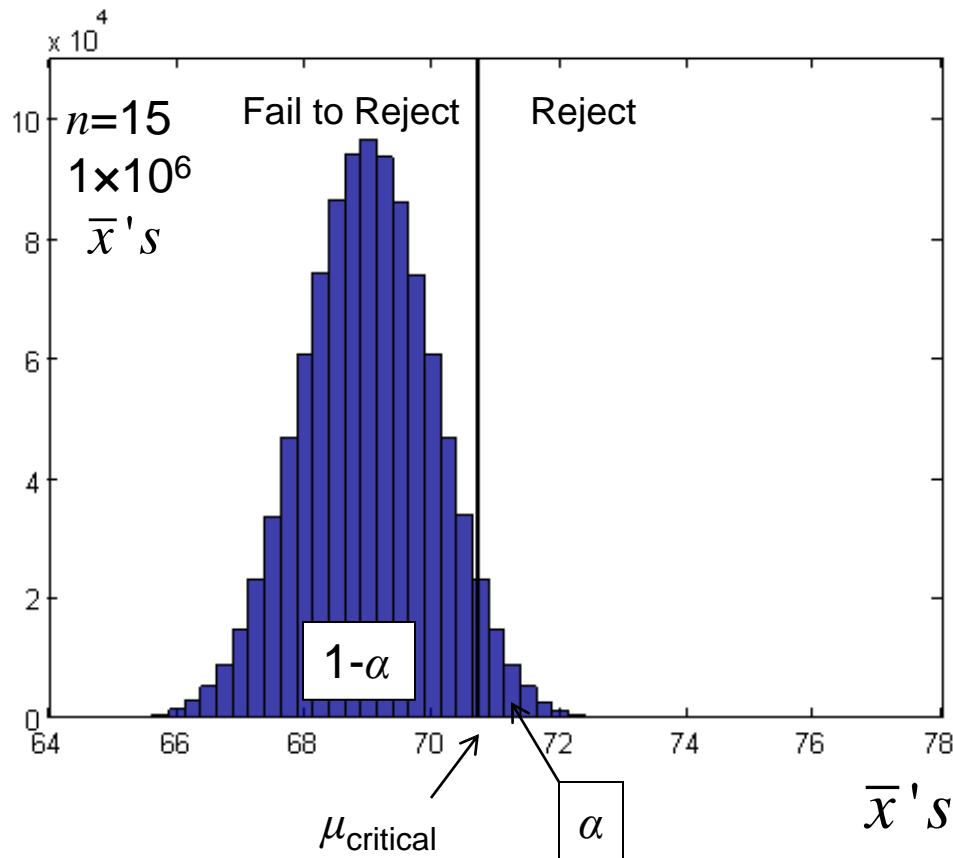


Calculated  
 $1 \times 10^6$  means with  $n=15$ .  
(Will repeat for  $\mu = 72''$ .)



# 8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach



$$H_0: \mu \leq 69'' \text{ vs. } H_a: \mu > 69''$$

$$\alpha = .05$$

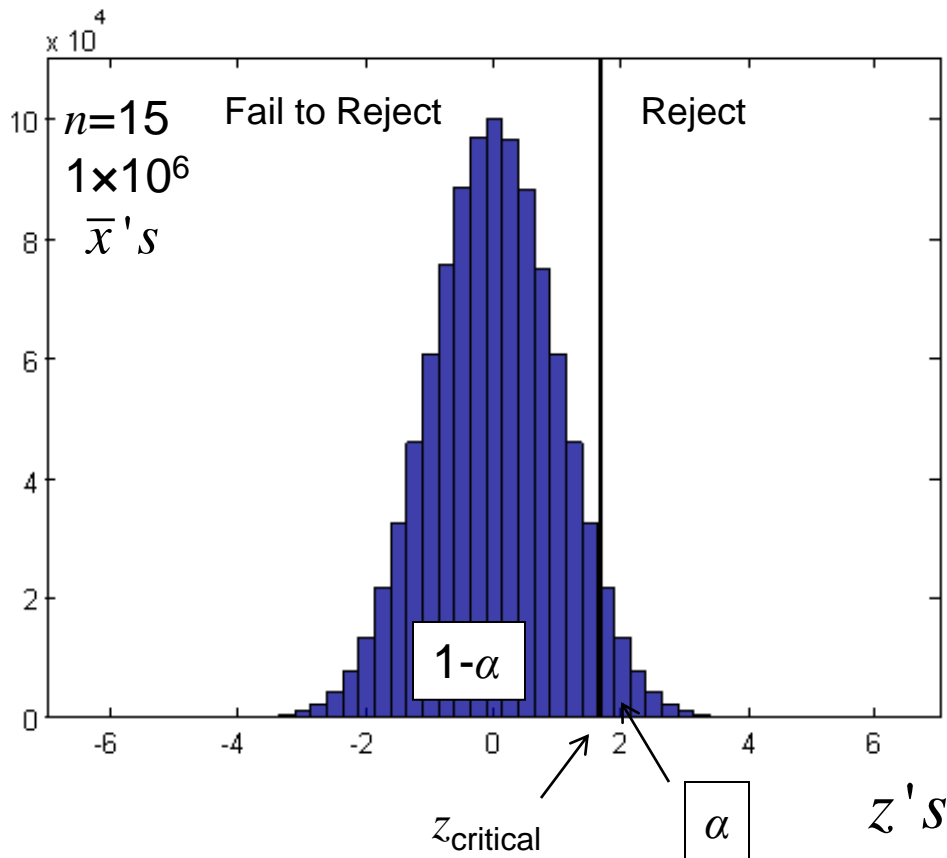
When the true mean  $\mu = 69''$ , we reject  $H_0$   $\alpha$  fraction of the time.

Commit a Type I Error.

Given  $\alpha$ , we want  $\mu_{critical}$ .

# 8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach



Instead of  $\mu_{critical}$  we find critical  $z$ ,  $z_{critical} = z(\alpha)$ .

Do this by assuming that  $H_0: \mu = 69$  is true, then calculate

$$z = \frac{\bar{x} - 69}{4 / \sqrt{15}}$$

# 8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

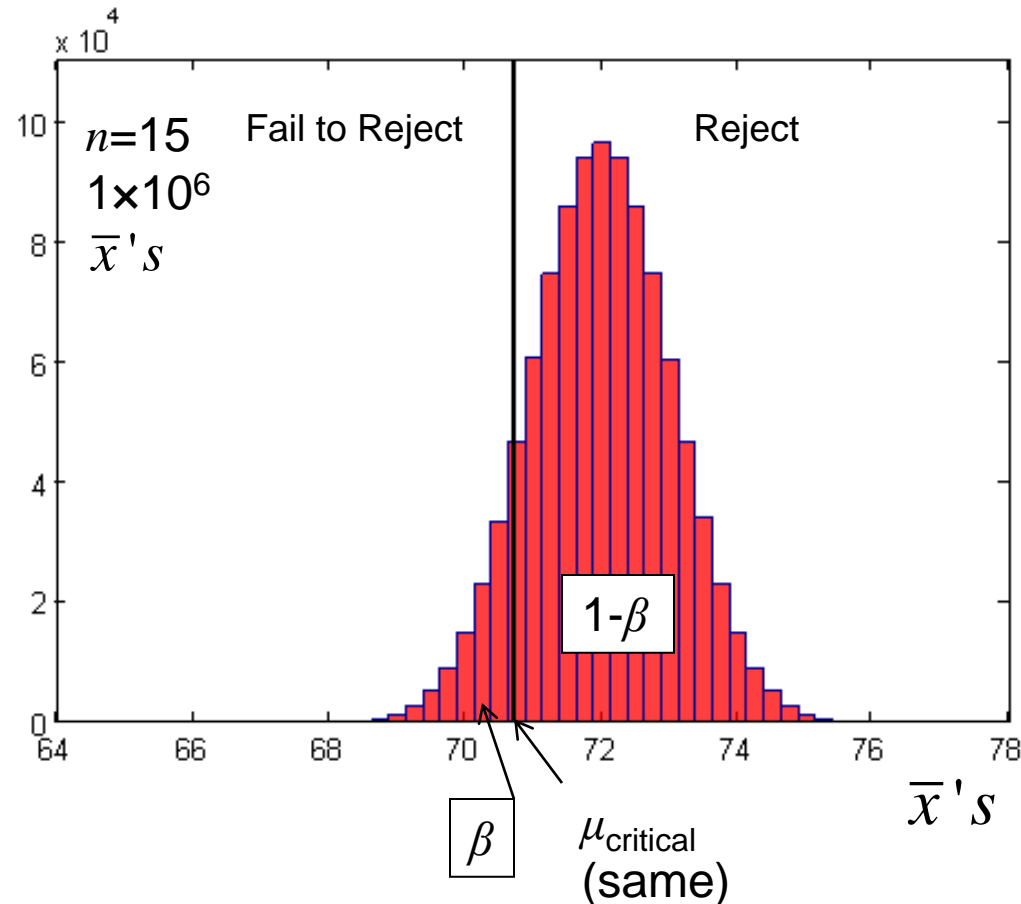
## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

$$H_0: \mu \leq 69 \text{ vs. } H_a: \mu > 69$$

$$\alpha = .05$$

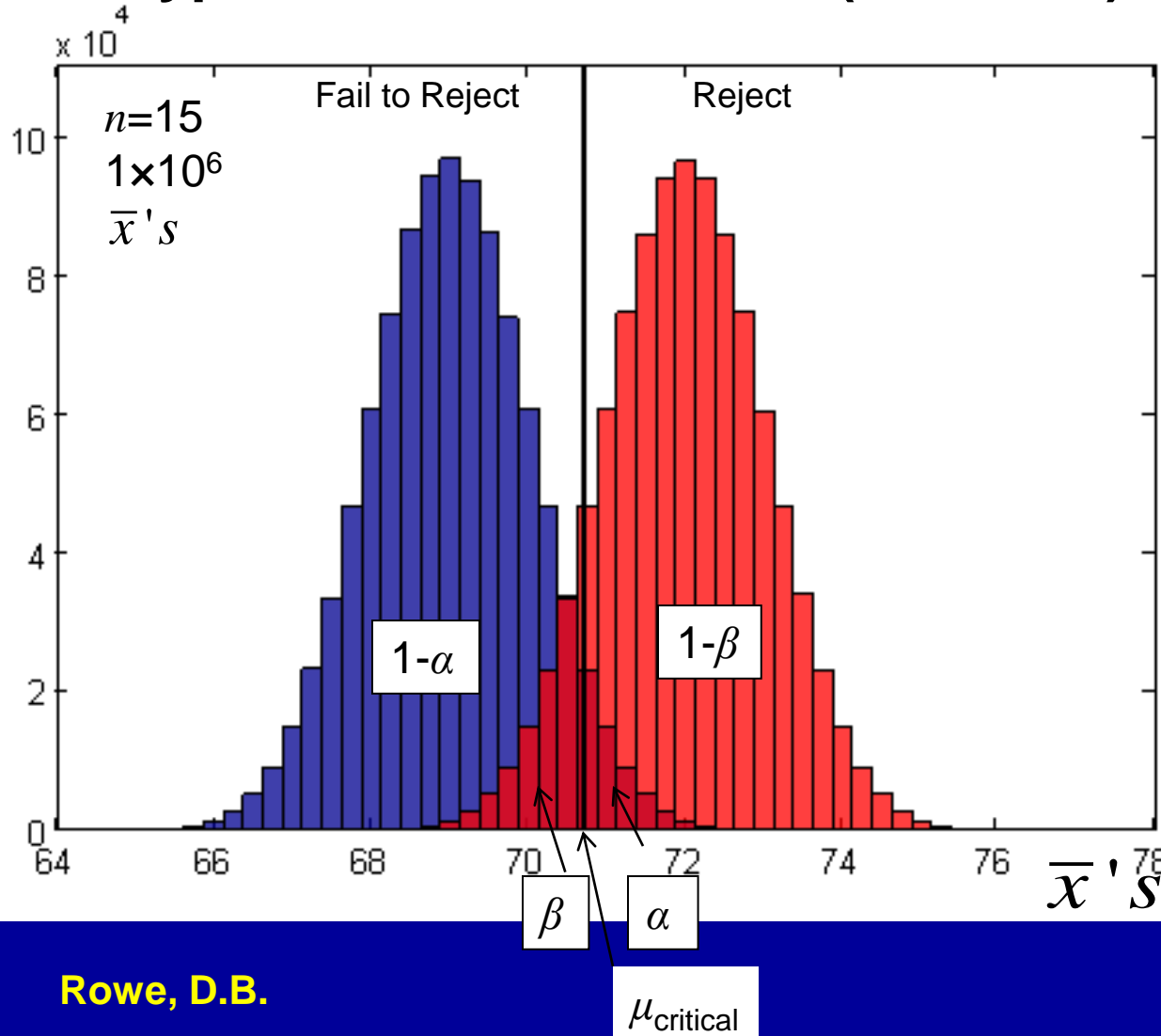
When the true mean  $\mu = 72$ ",  
we do not reject  $H_0$   $\beta$   
fraction of the time.

Commit a Type II Error



# 8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach



	$H_0$ True ( $\mu=69''$ )	$H_0$ False ( $\mu=72''$ )
Fail to Reject $H_0$	Correct Decision ( $1-\alpha$ )	Type II Error ( $\beta$ )
Reject $H_0$	Type I Error ( $\alpha$ )	Correct Decision ( $1-\beta$ )

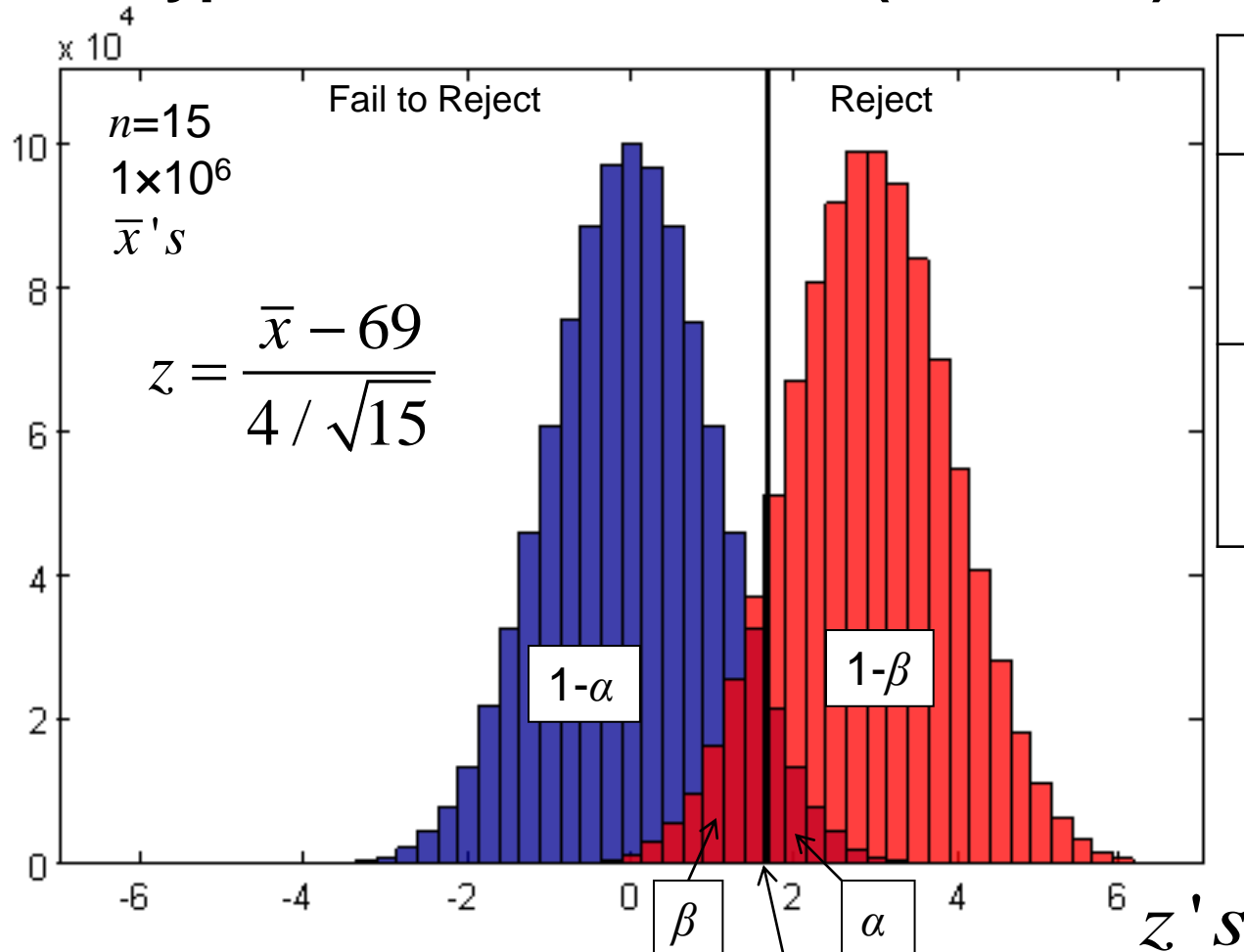
Power of the test:  
 $1 - \beta = P(\text{Reject } H_0 \mid H_0 \text{ False})$

Discrimination ability.  
 Ability to detect difference.



# 8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach



	$H_0$ True ( $\mu=69''$ )	$H_0$ False ( $\mu=72''$ )
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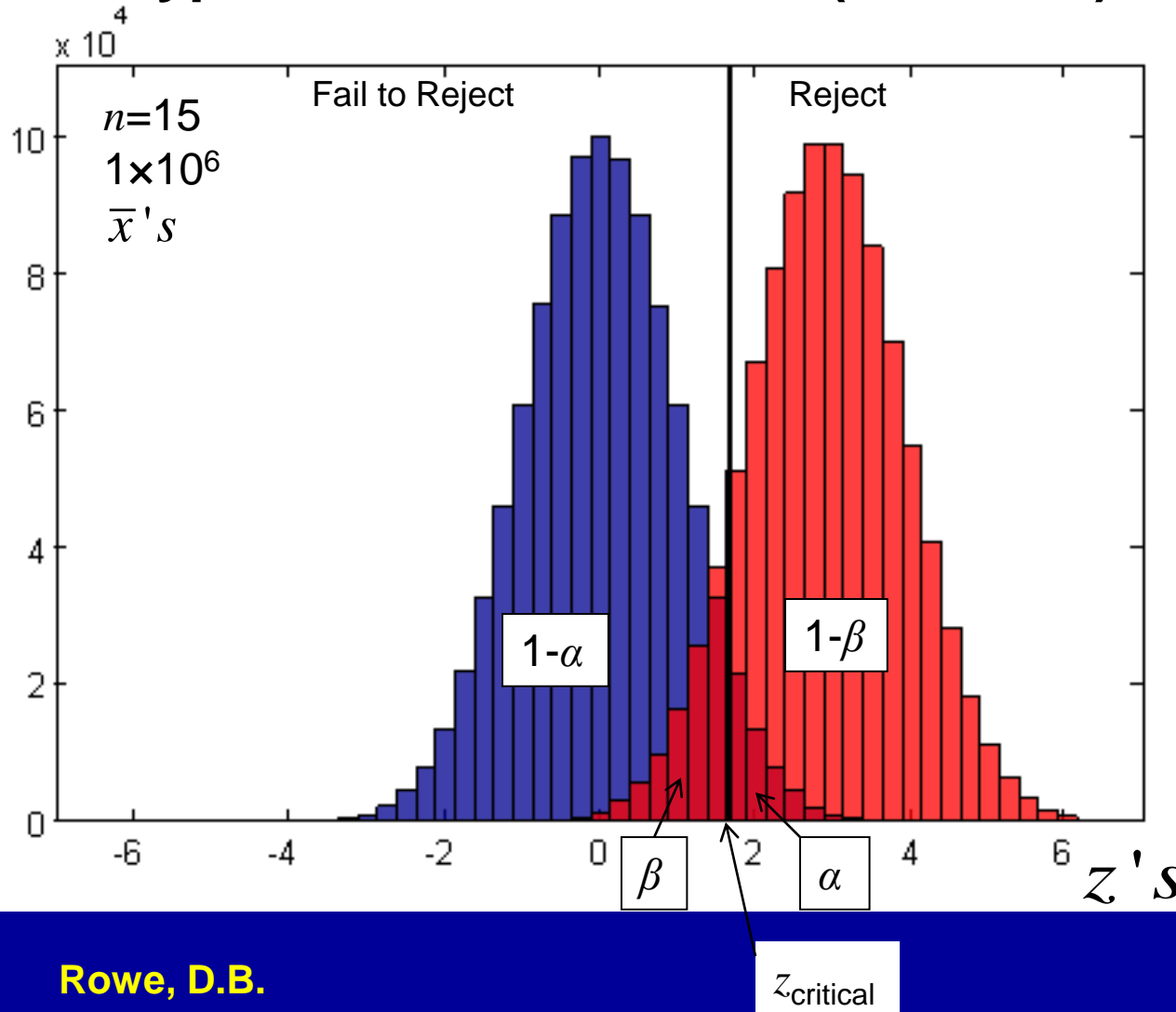
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# 8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach



We want our  $\alpha$ , Prob of Type I Error to be small.

So why not just decrease  $\alpha$ ?

Decreasing  $\alpha$  increases  $\beta$ .

And vice versa.

# 8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

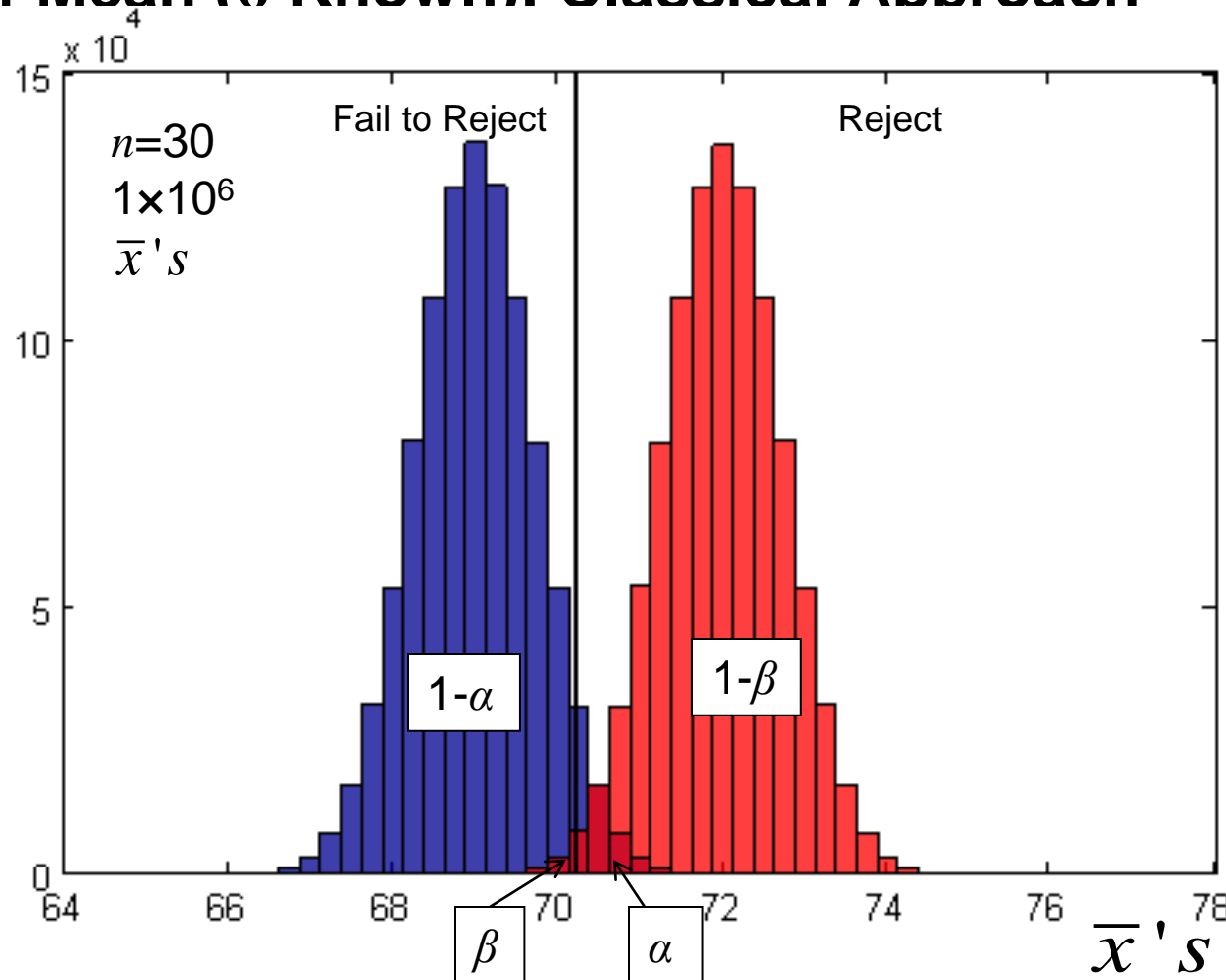
## 8.5 Hypothesis Test of Mean ( $\sigma$ Known): Classical Approach

What is the solution?

Increase  $n$ .

Figure shows  $n$   
increased to  $n = 30$   
from  $n = 15$ .

Note  $\alpha$  and  $\beta$  both  
smaller with larger  $n$ .



# 8: Introduction to Statistical Inference $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

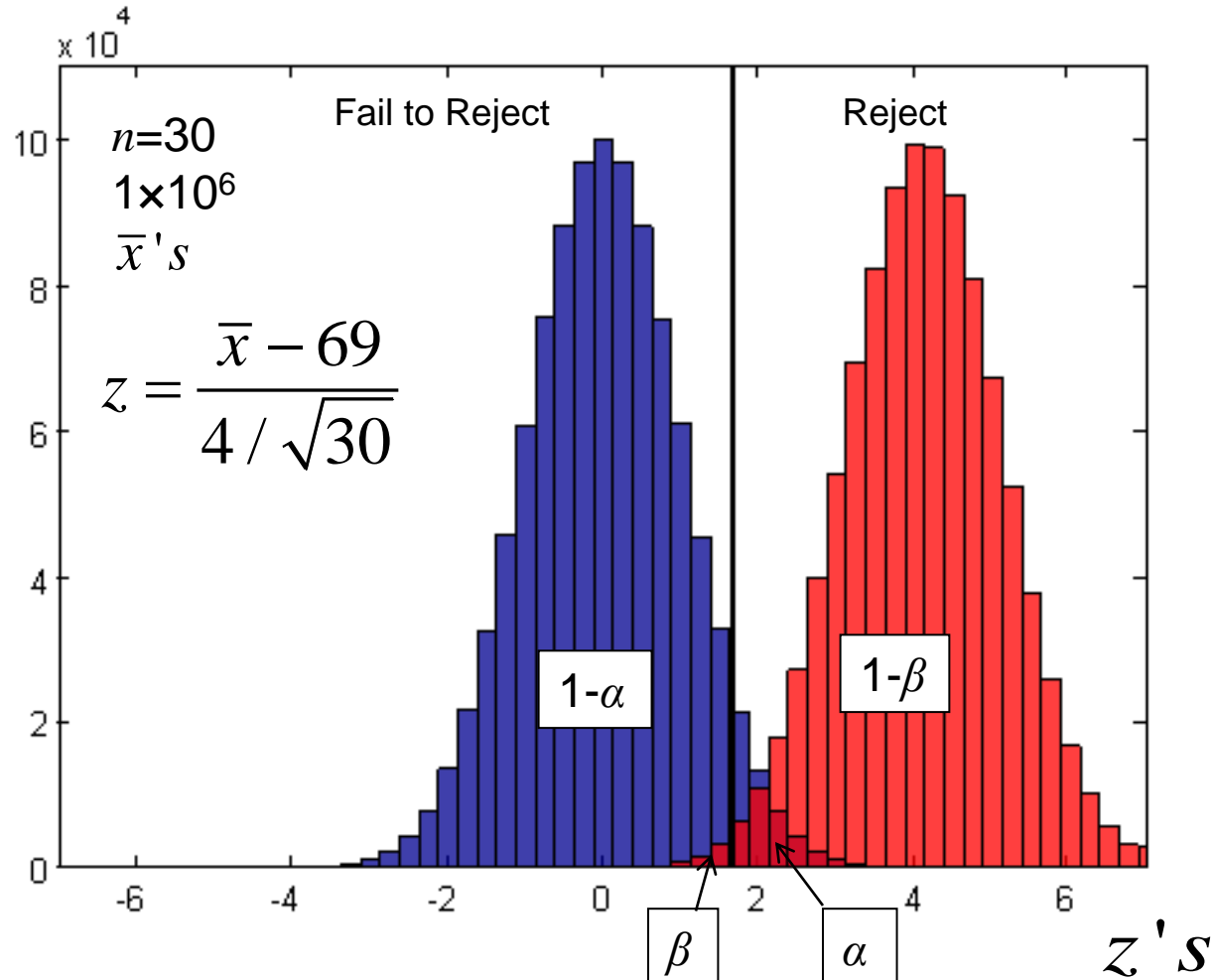
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# Chapter 8: Introduction to Statistical Inference

Questions?

Homework: Read 8.3-8.5

WebAssign

Chapter 8 # 57, 59, 81, 93, 97, 106, 109,  
119, 145, 157