Class 6

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Agenda:

Recap Chapter 4.1, 4.2

Lecture Chapter 4.3 - 4.5

Recap Chapter 4.1, 4.2

4: Probability

4.1 Probability of Events

An **experiment** is a process by which a measurement is taken or observations is made. i.e. *flip coin* or *roll die*

An **outcome** is the result of an experiment. i.e. *Heads*, or *3*

Sample space is a listing of possible outcomes. i.e. $S=\{H,T\}$

An **event** is an outcome or a combination of outcomes.
i.e. A=even number when rolling a die=$\{2,4,6\}$

4: Probability

4.1 Probability of Events

A_i are events

Property 1: $0 \le P(A_i) \le 1$

Property 2: $\sum_{i=1}^{n} P(O_i) = 1$

$i = 1, \ldots, n$

O_i are outcomes

Approaches to probability.

(1) Empirical (AKA experimental)

$$\text{empirical probability of } A = \frac{\text{number of times } A \text{ occured}}{\text{number of trials}}$$

(2) Theoretical (AKA classical or equally likely)

$$\text{theoretical probability of } A = \frac{\text{number of times } A \text{ occus in sample space}}{\text{number of elements in the sample space}}$$

4: Probability - Empirical

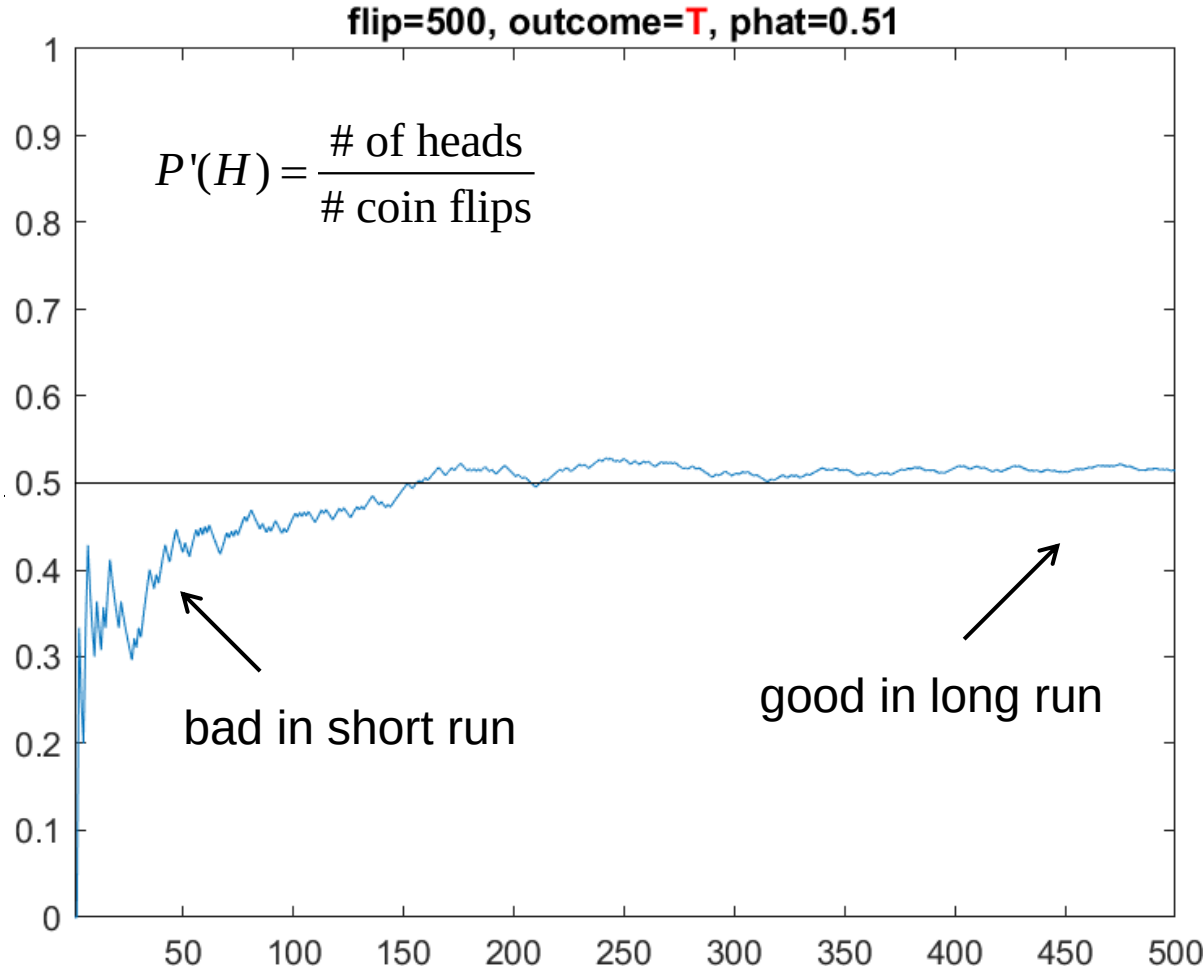
4.1 Probability of Events – Law of large numbers

Had computer flip a single coin 1000 times.

Flip # on x axis

$P'(H)$ on y axis.

This shows convergence to true value of 1/2.

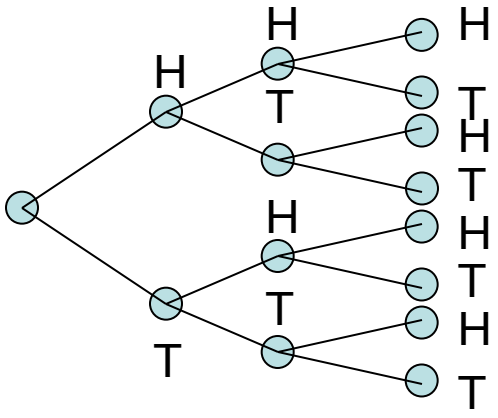


4: Probability - Theoretical

4.1 Probability of Events

So let's flip a coin twice.

Can flip three times.



Sample space:
listing of outcomes
for 3 flips

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$$P(HHH) = \frac{\#\text{ times } HHH \text{ occurs in } S}{\#\text{ elements in } S}$$

Chapter 4: Probability continued

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Children's book

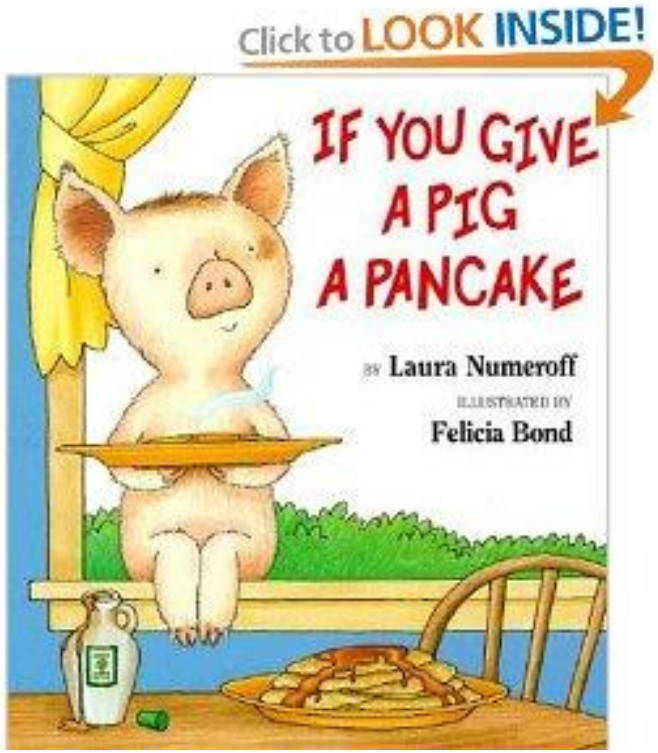
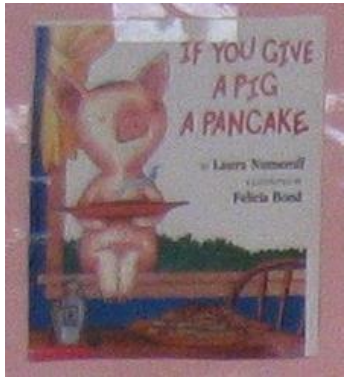
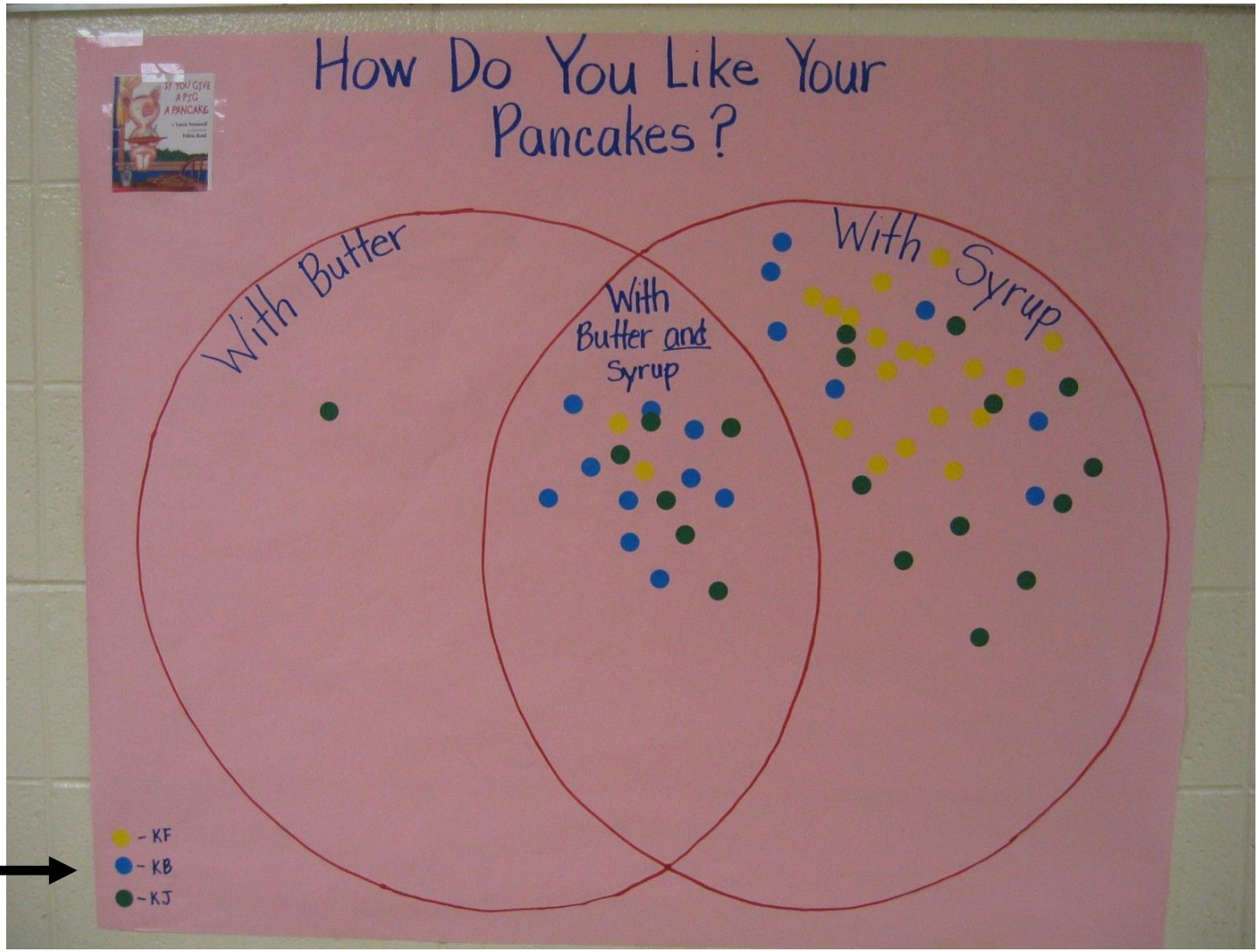


Figure from amazon.com

Venn Diagram



3 kindergarten classes



4: Probability

4.2 Rules of Probability – Probability of "A or B"

General Addition Rule

Let A and B be two events defined in the sample space, S.

In words: probability of A or B = probability of A
+ probability of B
- probability of A and B

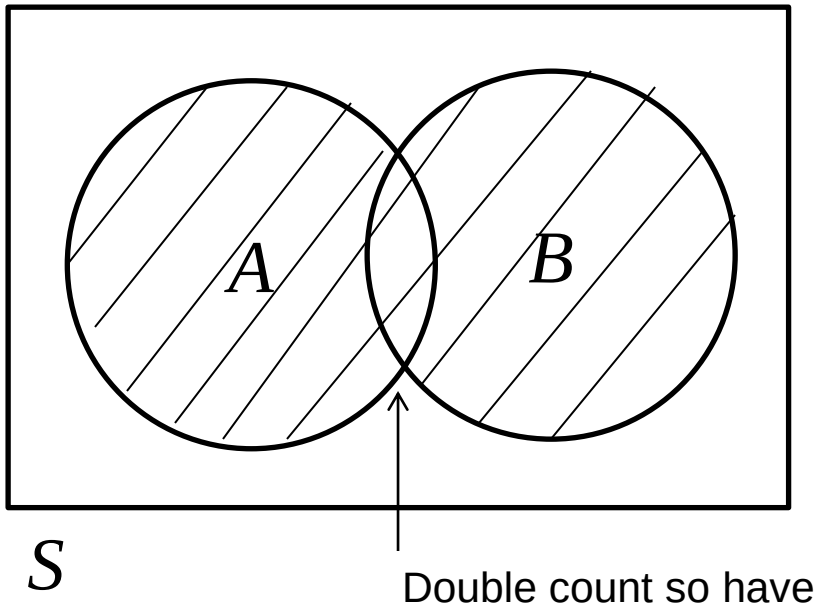
In algebra: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ (4.4)

4: Probability

4.2 Rules of Probability – Probability of "A or B"

Union: A or B

Venn Diagram:



$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Double count so have to subtract one off.

4: Probability

4.2 Rules of Probability – "A or B"

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example: Pick Card, A=Heart, B=Ace

P(Heart or Ace)

P(Heart) $=$

Figure from Johnson & Kuby, 2012.

4: Probability

4.2 Rules of Probability – "A or B"

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example: Pick Card, A=Heart, B=Ace

P(Heart or Ace)

$P(\text{Heart}) = 13/52$

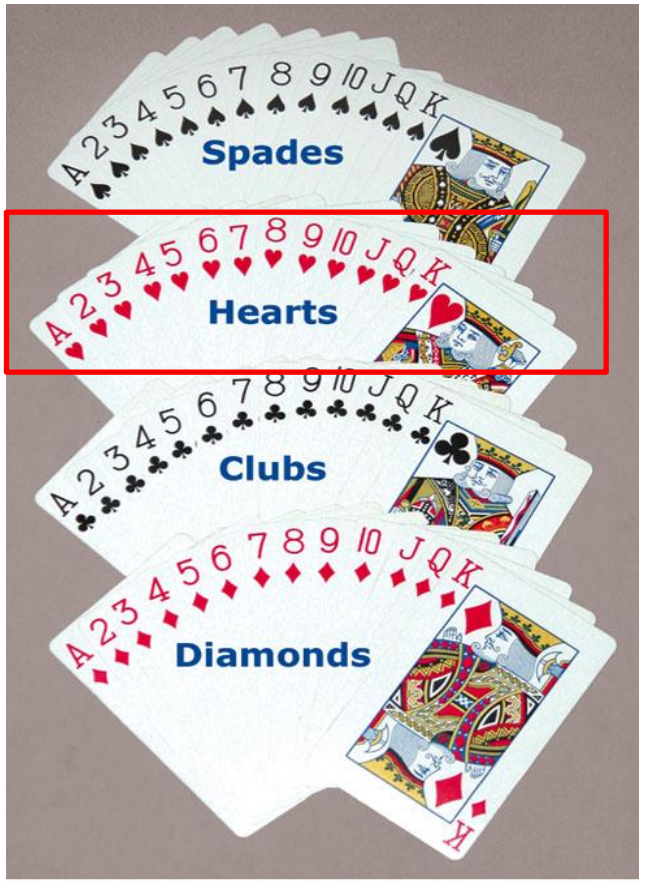


Figure from Johnson & Kuby, 2012.

4: Probability

4.2 Rules of Probability – "A or B"

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example: Pick Card, A=Heart, B=Ace

P(Heart or Ace)

P(Ace) $=$

Figure from Johnson & Kuby, 2012.

4: Probability

4.2 Rules of Probability – "A or B"

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example: Pick Card, A=Heart, B=Ace

P(Heart or Ace)

$P(\text{Ace}) = 4/52$

Figure from Johnson & Kuby, 2012.

4: Probability

4.2 Rules of Probability – "A or B"

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example: Pick Card, A=Heart, B=Ace

P(Heart or Ace)

P(Heart and Ace) $=$

Figure from Johnson & Kuby, 2012.

4: Probability

4.2 Rules of Probability – "A or B"

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example: Pick Card, A=Heart, B=Ace

P(Heart or Ace)

P(Heart and Ace) $= 1/52$

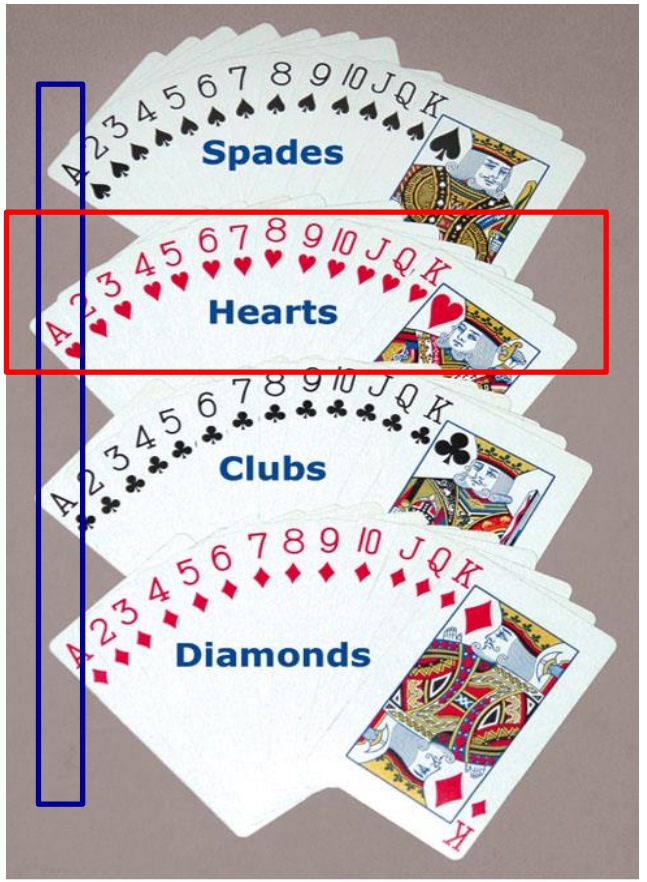


Figure from Johnson & Kuby, 2012.

4: Probability

4.2 Rules of Probability – "A or B"

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example: Pick Card, A=Heart, B=Ace

P(Heart or Ace)

P(Heart) $=$

P(Ace) $=$

P(Heart and Ace) $=$

Figure from Johnson & Kuby, 2012.

$$P(\text{Heart or Ace}) = P(\text{Heart}) + P(\text{Ace}) - P(\text{Heart and Ace})$$

4: Probability

4.2 Rules of Probability – "A or B"

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example: Pick Card, A=Heart, B=Ace

$P(\text{Heart or Ace})$

$P(\text{Heart}) = 13/52$

$P(\text{Ace}) = 4/52$

$P(\text{Heart and Ace}) = 1/52$

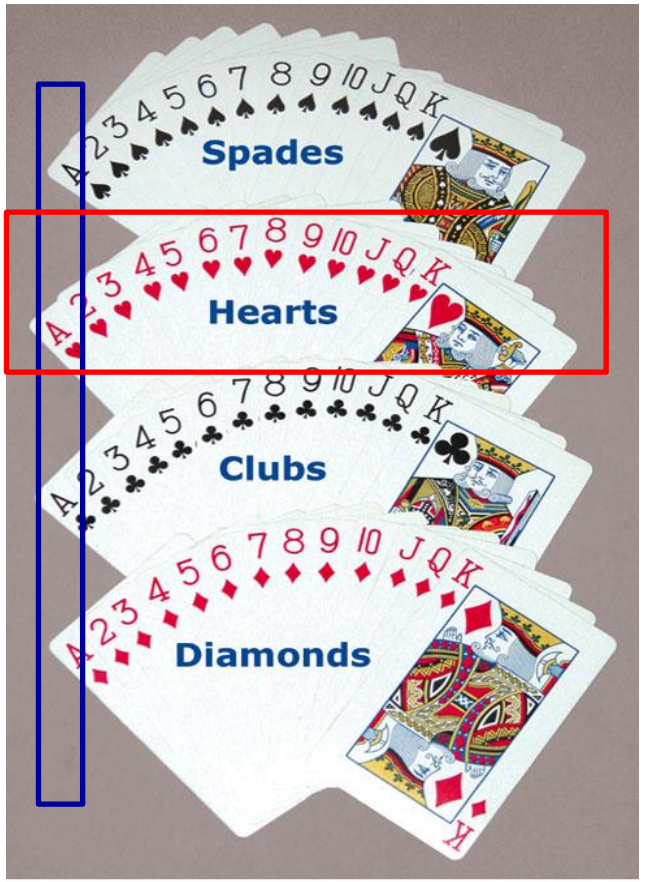


Figure from Johnson & Kuby, 2012.

$$P(\text{Heart or Ace}) = P(\text{Heart}) + P(\text{Ace}) - P(\text{Heart and Ace})$$

$$P(\text{Heart or Ace}) = 13/52 + 4/52 - 1/52 = 16/52$$

4: Probability

4.3 Rules of Probability – Probability of "Not A"

Complimentary Events: The *compliment of* A, $\bar{A}$ is the set of all sample points in the sample space that does not belong to event A. i.e. If A, is heads, then $\bar{A}$ is tails.

Compliment Rule:
In words: probability of A compliment = one – probability of A

In algebra: $P(\bar{A}) = 1 - P(A)$

From $P(A) + P(\bar{A}) = 1$ (4.3)

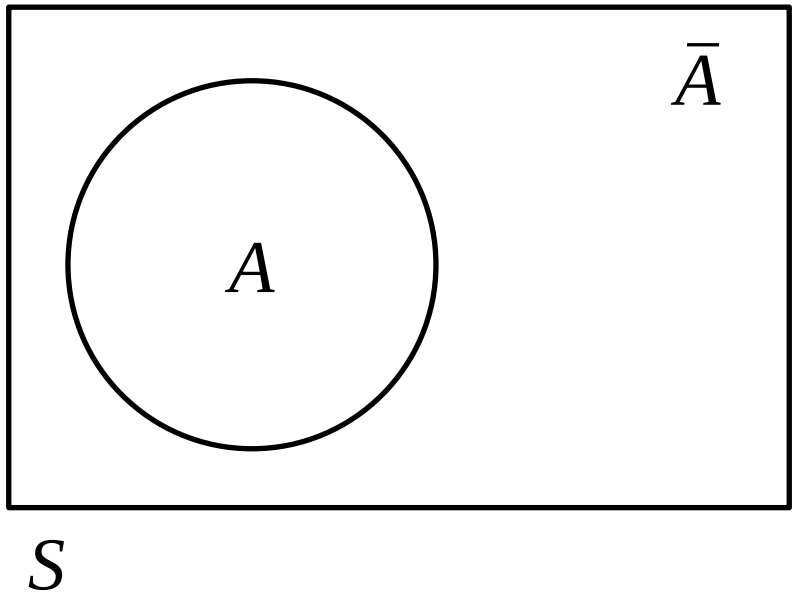
i.e. $P(T) = 1 - P(H)$

4: Probability

4.3 Rules of Probability – Probability of "Not A"

Compliment: $S = \{A, \bar{A}\}$

Venn Diagram:



$$P(\bar{A}) = 1 - P(A)$$

4: Probability

4.3 Rules of Probability – Probability of "A and B"

General Multiplication Rule

Let A and B be two events defined in the sample space, S.

In words: probability of A and B = probability of A
× probability of B, knowing A

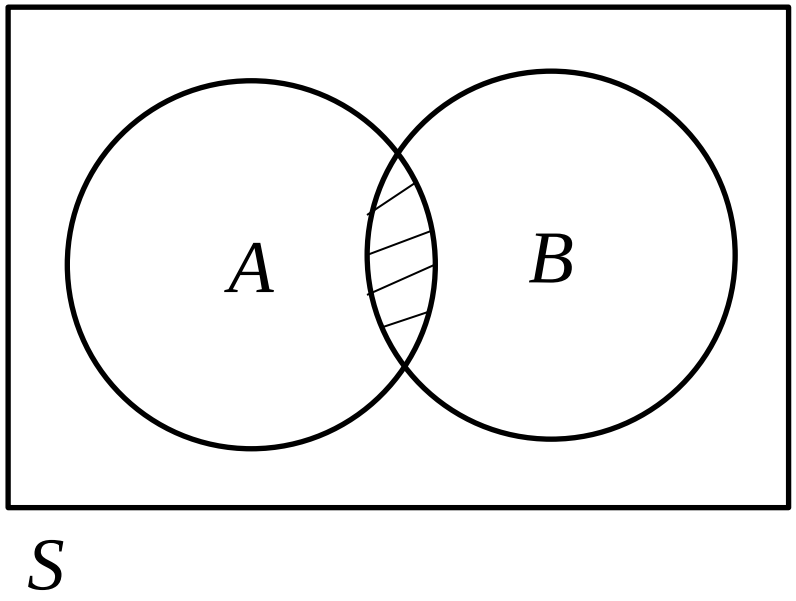
In algebra: $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ (4.5)

4: Probability

4.3 Rules of Probability – Probability of "A and B"

Event Intersection: A and B

Venn Diagram:



$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

4: Probability

4.3 Rules of Probability – "A and B"

$$P(A \text{ and } B) = P(B)P(A \mid B)$$

Example: Pick Card, A=Heart, B=Ace

P(Heart and Ace)

P(Ace) =

Figure from Johnson & Kuby, 2012.

4: Probability

4.3 Rules of Probability – "A and B"

$$P(A \text{ and } B) = P(B)P(A \mid B)$$

Example: Pick Card, A=Heart, B=Ace

P(Heart and Ace)

$P(\text{Ace}) = 4/52$

Figure from Johnson & Kuby, 2012.

4: Probability

4.3 Rules of Probability – "A and B"

$$P(A \text{ and } B) = P(B)P(A \mid B)$$

Example: Pick Card, A=Heart, B=Ace

P(Heart and Ace)

$P(\text{Heart} \mid Ace) =$

Figure from Johnson & Kuby, 2012.

4: Probability

4.3 Rules of Probability – "*A* and *B*"

$$P(A \text{ and } B) = P(B)P(A \mid B)$$

Example: Pick Card, A=Heart, B=Ace

P(Heart and Ace)

$P(\text{Heart} \mid Ace) = 1/4$

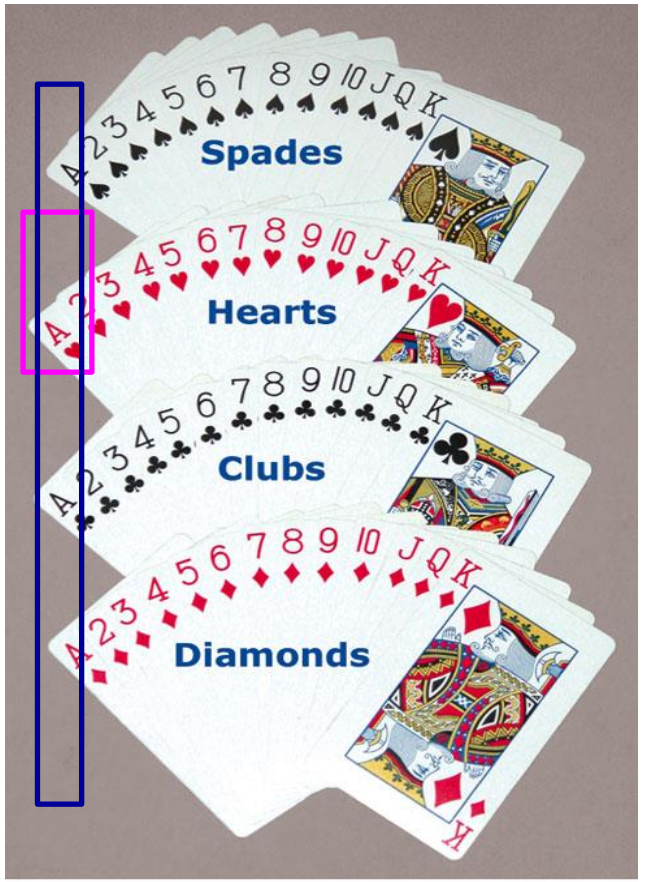


Figure from Johnson & Kuby, 2012.

4: Probability

4.3 Rules of Probability – "A and B"

$$P(A \text{ and } B) = P(B)P(A \mid B)$$

Example: Pick Card, A=Heart, B=Ace

$P(\text{Heart and Ace})$

$P(\text{Ace}) =$

$P(\text{Heart} \mid Ace) =$

$$P(\text{Heart and Ace}) = P(\text{Ace})P(\text{Heart}|\text{Ace})$$

Figure from Johnson & Kuby, 2012.

4: Probability

4.3 Rules of Probability – "A and B"

$$P(A \text{ and } B) = P(B)P(A \mid B)$$

Example: Pick Card, A=Heart, B=Ace

$P(\text{Heart and Ace})$

$P(\text{Ace}) = 4/52$

$P(\text{Heart} \mid Ace) = 1/4$

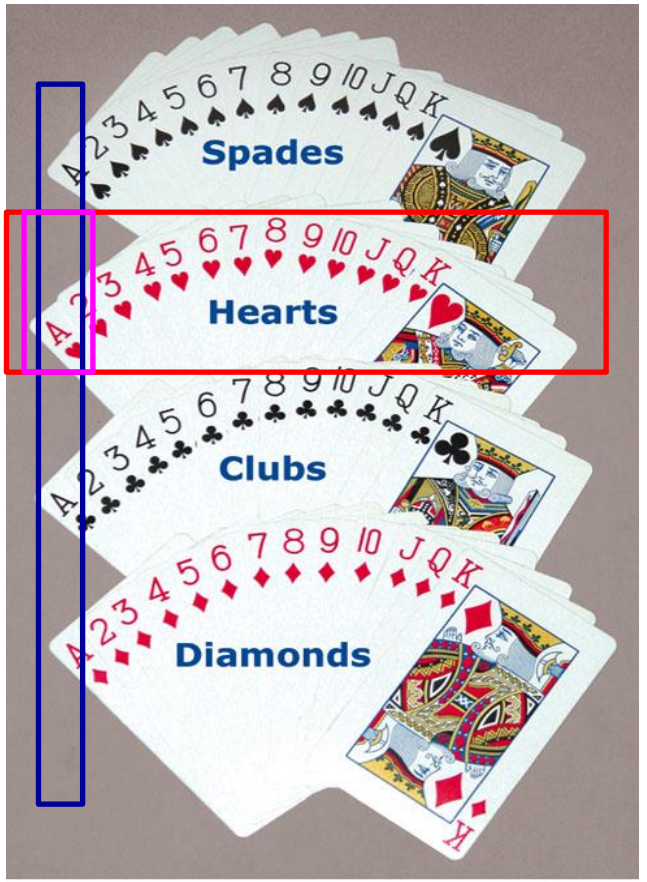


Figure from Johnson & Kuby, 2012.

$P(\text{Heart and Ace}) = P(\text{Ace})P(\text{Heart}|\text{Ace})$

$P(\text{Heart and Ace}) = (4/52)(1/4) = 1/52$

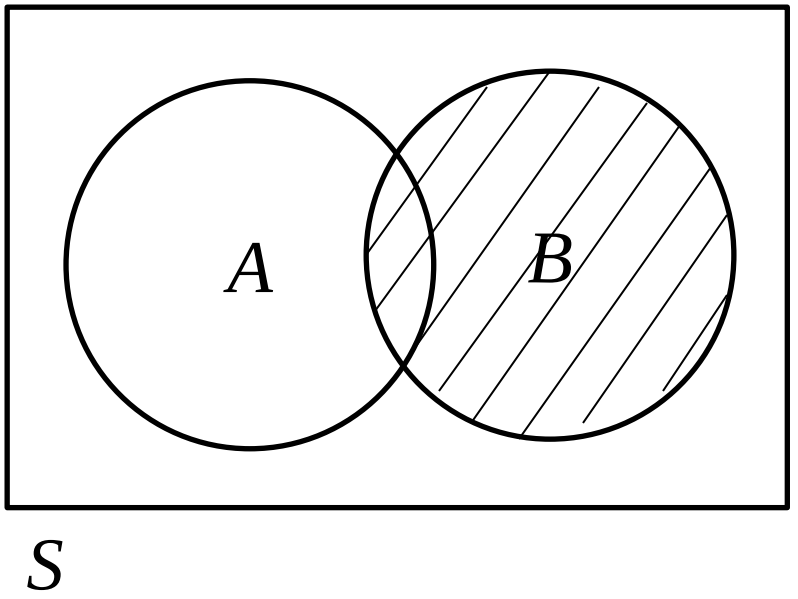
4: Probability

4.3 Rules of Probability – Probability of "A and B"

Conditional Probability: Probability of event A given that event B has occurred is

$$P(A \mid B) = \frac{\mathrm{P}(A \text{ and } B)}{\mathrm{P}(B)}$$

the "|" is spoken as "given"
or "knowing"



4: Probability

4.3 Rules of Probability – "*A* and *B*"

$$P(A \mid B) = \frac{\mathrm{P}(A \text{ and } B)}{\mathrm{P}(B)}$$

Example: Pick Card, A=Heart, B=Ace

$P(\text{Heart} \mid Ace)$

$P(\text{Heart and Ace}) =$

Figure from Johnson & Kuby, 2012.

4: Probability

4.3 Rules of Probability – "*A* and *B*"

$$P(A \mid B) = \frac{\mathrm{P}(A \text{ and } B)}{\mathrm{P}(B)}$$

Example: Pick Card, A=Heart, B=Ace

$P(\text{Heart} \mid Ace)$

$P(\text{Heart and Ace}) = 1/52$

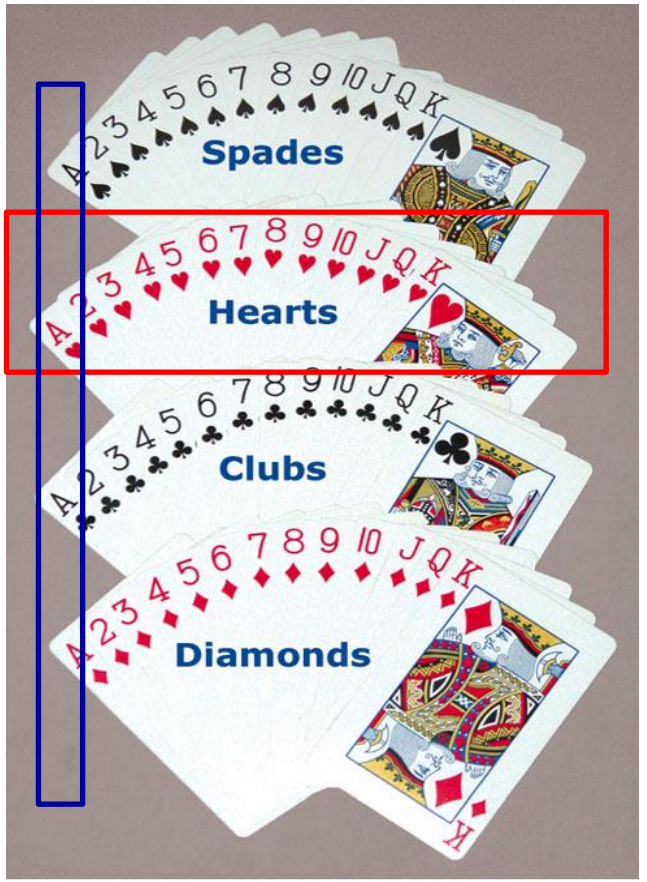


Figure from Johnson & Kuby, 2012.

4: Probability

4.3 Rules of Probability – "A and B"

$$P(A \mid B) = \frac{\mathrm{P}(A \text{ and } B)}{\mathrm{P}(B)}$$

Example: Pick Card, A=Heart, B=Ace

$P(\text{Heart} \mid Ace)$

$P(\text{Ace}) =$

Figure from Johnson & Kuby, 2012.

4: Probability

4.3 Rules of Probability – "*A* and *B*"

$$P(A \mid B) = \frac{\mathrm{P}(A \text{ and } B)}{\mathrm{P}(B)}$$

Example: Pick Card, A=Heart, B=Ace

$P(\text{Heart} \mid Ace)$

$P(\text{Ace}) = 4/52$

Figure from Johnson & Kuby, 2012.

4: Probability

4.3 Rules of Probability – "A and B"

$$P(A \mid B) = \frac{\mathrm{P}(A \text{ and } B)}{\mathrm{P}(B)}$$

Example: Pick Card, A=Heart, B=Ace

$P(\text{Heart} \mid Ace)$

$$P(\text{Heart} \mid Ace) = \frac{P(\text{Heart and Ace})}{P(Ace)}$$

$P(\text{Heart} \mid Ace) =$

Figure from Johnson & Kuby, 2012.

4: Probability

4.3 Rules of Probability – "A and B"

$$P(A \mid B) = \frac{\mathrm{P}(A \text{ and } B)}{\mathrm{P}(B)}$$

Example: Pick Card, A=Heart, B=Ace

$P(\text{Heart} \mid Ace)$

$P(\text{Heart and Ace}) = 1/52$

$P(\text{Ace}) = 4/52$

$$P(\text{Heart} \mid Ace) = \frac{P(\text{Heart and Ace})}{P(Ace)}$$

$P(\text{Heart} \mid Ace) = (1/52)/(4/52) = 1/4$

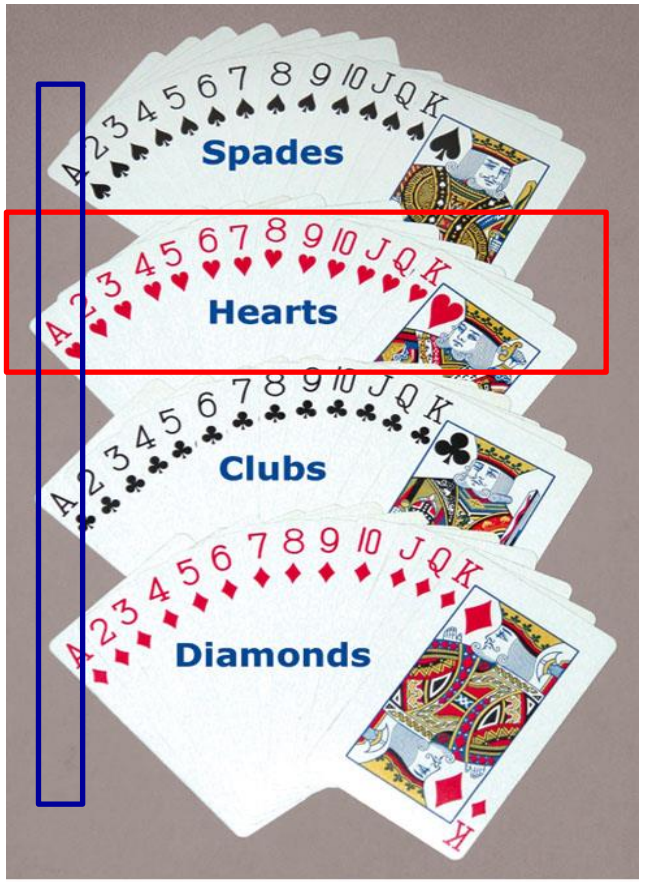
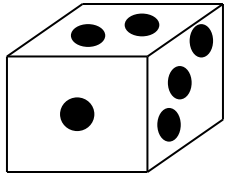
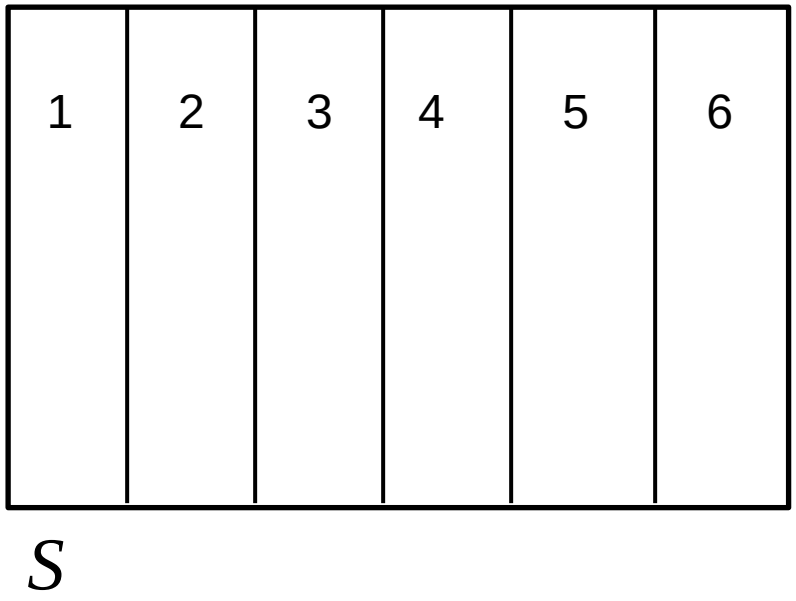


Figure from Johnson & Kuby, 2012.

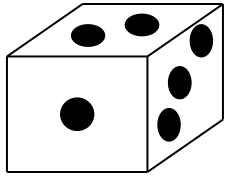
4: Probability
4.3 Rules of Probability



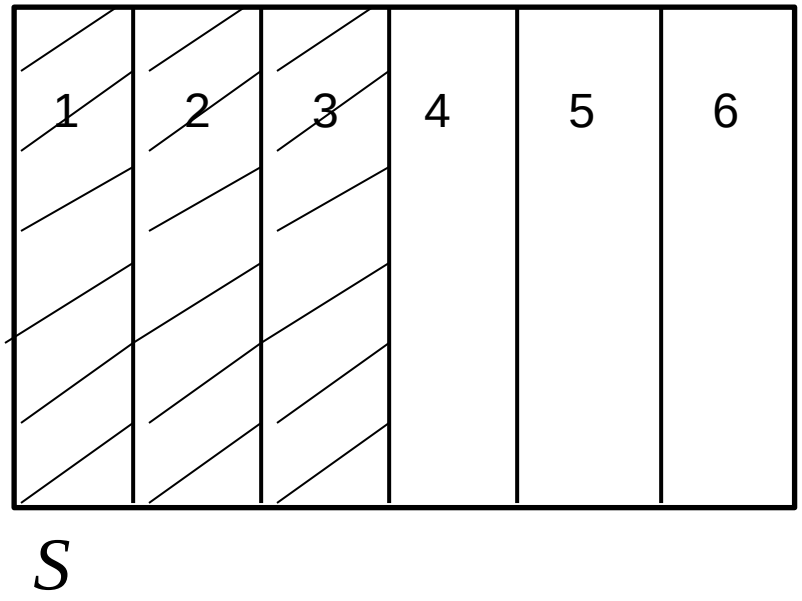
Union Example (A or B): Rolling a single die.
A=event #1,2,3. B=event odd number.



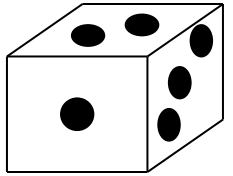
4: Probability
4.3 Rules of Probability



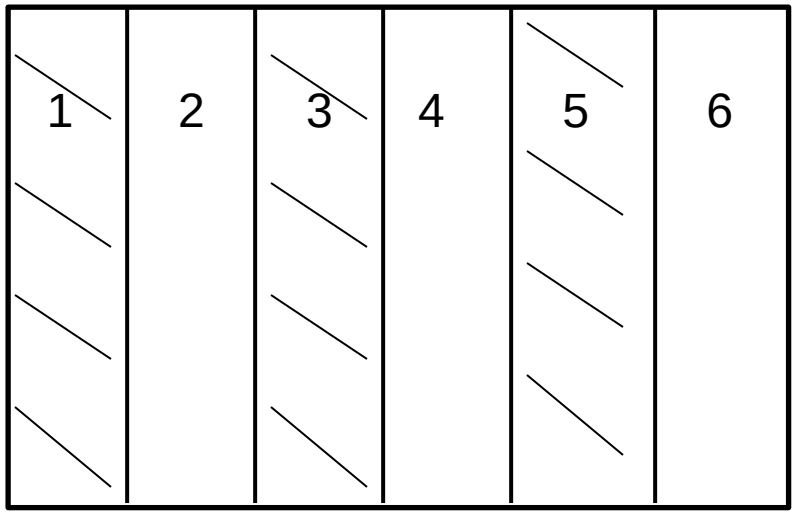
Union Example (A or B): Rolling a single die.
A=event #1,2,3. B=event odd number.
$A=\{1,2,3\}$



4: Probability
4.3 Rules of Probability



Union Example (A or B): Rolling a single die.
A=event #1,2,3. B=event odd number.
$B=\{1,3,5\}$



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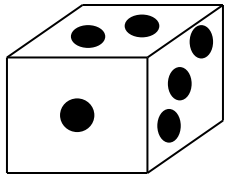
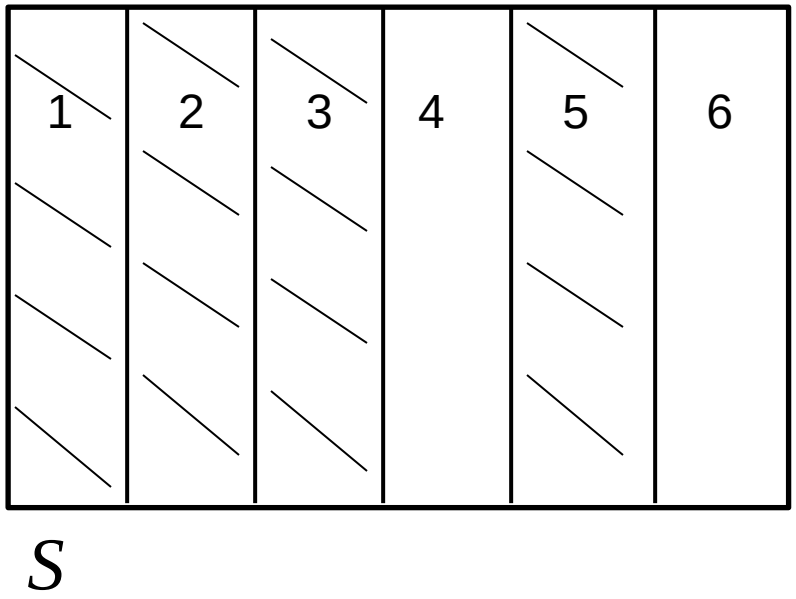
4: Probability
4.3 Rules of Probability

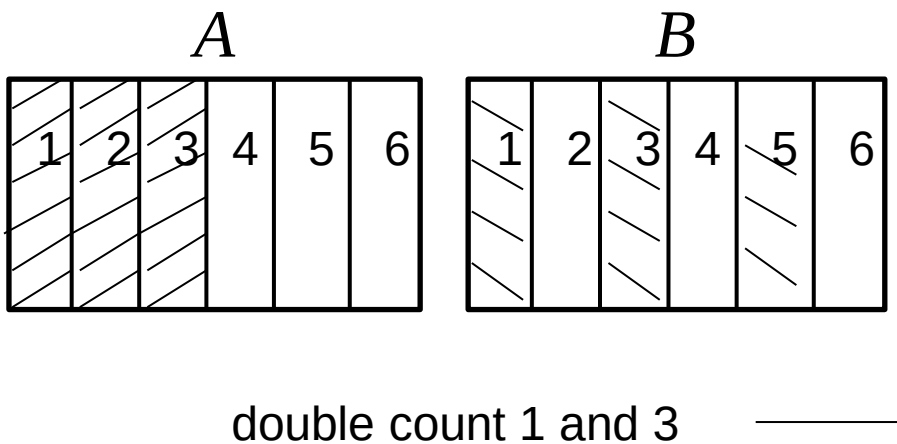


Union Example (A or B): Rolling a single die.
A=event #1,2,3. B=event odd number.

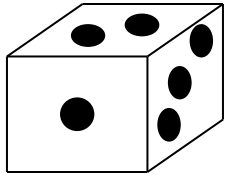
$(A \text{ or } B)$



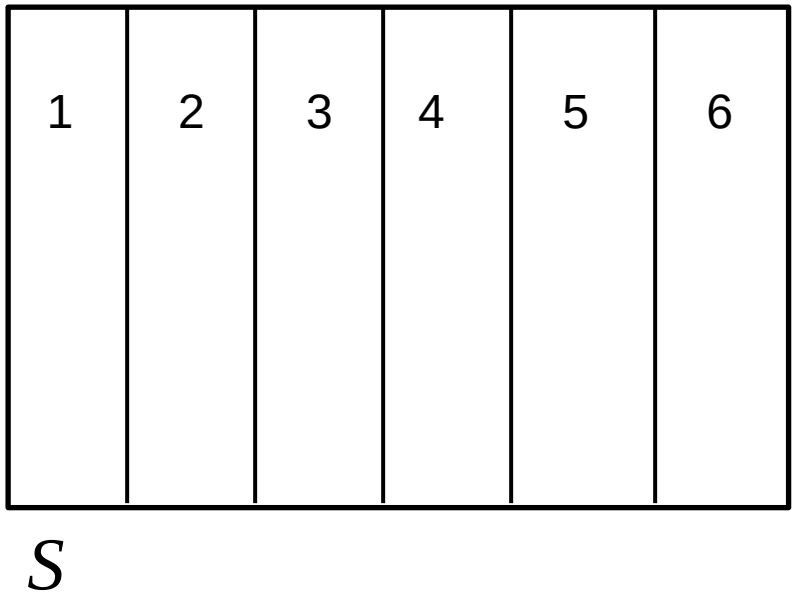
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



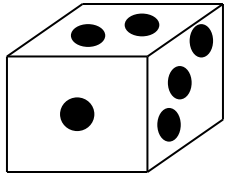
4: Probability
4.3 Rules of Probability



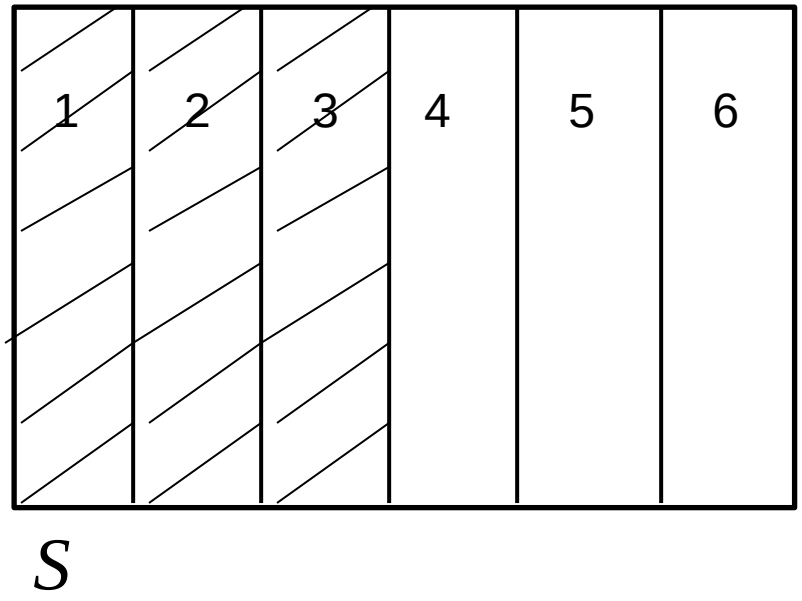
Intersection Example (A and B): Rolling a single die.
A=event #1,2,3. B=event odd number.



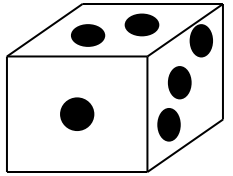
4: Probability
4.3 Rules of Probability



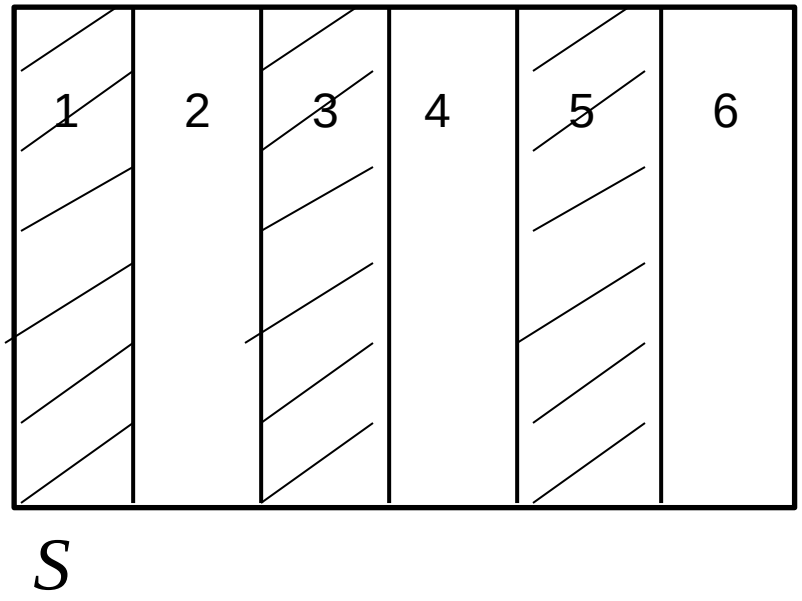
Intersection Example (A and B): Rolling a single die.
A=event #1,2,3. B=event odd number.
$A=\{1,2,3\}$



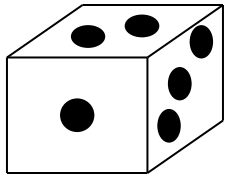
4: Probability
4.3 Rules of Probability



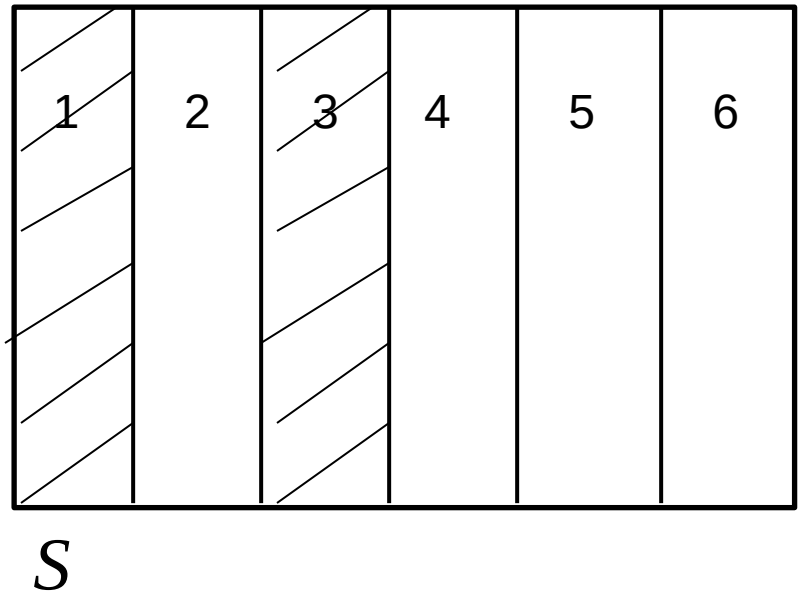
Intersection Example (A and B): Rolling a single die.
A=event #1,2,3. B=event odd number.
$B=\{1,3,5\}$



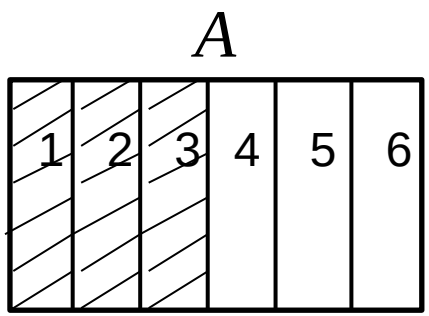
4: Probability
4.3 Rules of Probability

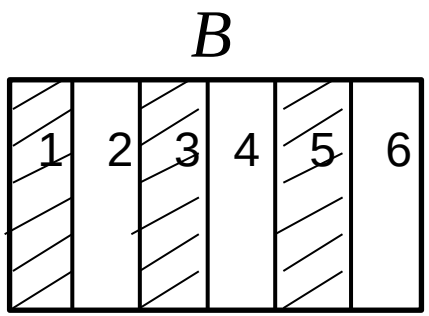


Intersection Example (A and B): Rolling a single die.
A=event #1,2,3. B=event odd number.
$(A \text{ and } B) = \{1,3\}$



$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$





4: Probability

4.4 Mutually Exclusive Events

Mutually exclusive events:
Events that share no common elements

In algebra: $P(A \text{ and } B) = 0$

In words:

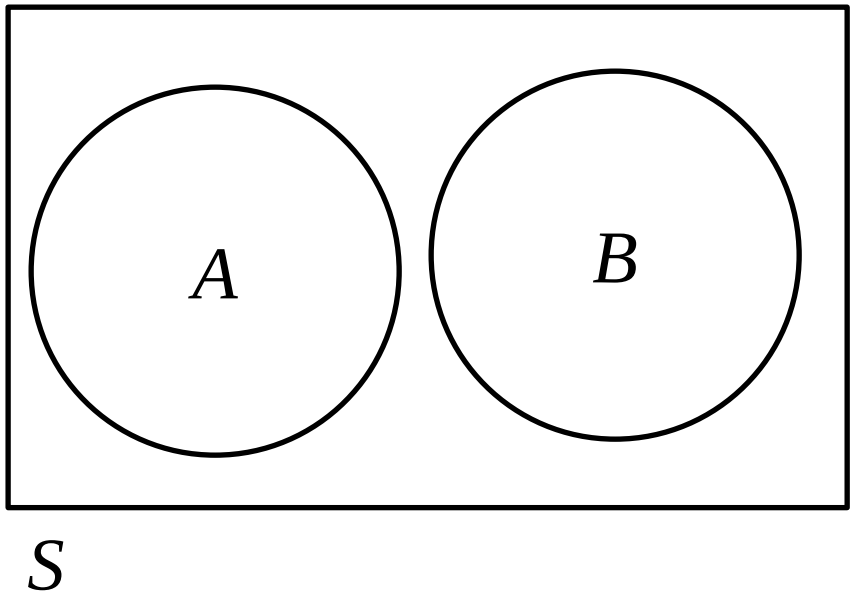
1. If one event has occurred, the other cannot.
2. None of the elements in one is in other.
3. In Venn diagrams, no intersection.
4. Intersection of events has a probability of zero.

4: Probability

4.4 Mutually Exclusive Events

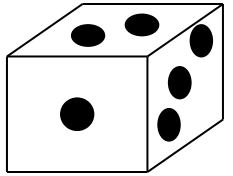
Mutually Exclusive: $P(A \text{ and } B) = 0$

Venn Diagram:

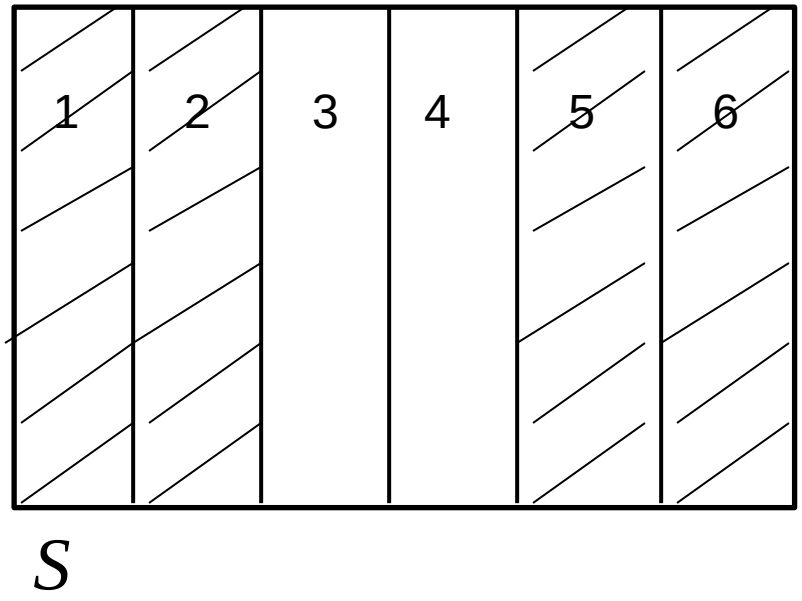


4: Probability

4.4 Mutually Exclusive Events



Mutually Exclusive Example: Rolling a single die.
A=event #1,2. B=event #5,6.



null set

$$A \text{ and } B = \varnothing$$

$$P(A \text{ and } B) = 0$$

4: Probability

4.5 Independent Events

Independent events: Two events are independent if the occurrence or nonoccurence of one gives us no information about the likeliness of occurrence for the other.

In algebra: $P(A) = P(A \mid B) = P(A \mid \text{not } B)$

In words:

1. Prob of A unaffected by knowledge that B has occurred, not occurred, or no knowledge.
2. …
3. …

4: Probability

4.5 Independent Events

Two events A and B are independent if the probability of one is not "influenced" by the occurrence or nonoccurrence of the other.

Two Events A and B are independent if:

1. $P(A) = P(A|B)$
2. $P(B) = P(B|A)$
3. $P(A \text{ and } B) = P(A) \cdot P(B)$

Examples:?

4: Probability
4.5 Independent Events

Dependent events: Events that are not independent. That is, occurrence of one event does have an effect on the probability of occurrence of the other event.

In algebra: $P(A) \neq P(A \mid B)$

4: Probability

4.5 Independent Events – Special multiplication rule

Special multiplication rule:
Let A and B be two independent events defined in a sample space S.

In words: The probability of A and B = probability of A × probability of B

In algebra: $P(A \text{ and } B) = P(A) \cdot P(B)$ (4.7)

More generally

$$P(A \text{ and } B \text{ and } C \text{ and } D \text{ and } E) = P(A) \cdot P(B) \cdot P(C) \cdot P(D) \cdot P(E)$$

4: Probability

Questions?

Homework: Read Chapter 4.3-4.5

Web Assign Chapter 4 # 59, 63, 65, 69, 85, 89, 91, 97, 105, 107, 113

Set one die to 4 (event B). Roll the other die 100 times. Let A be that a 3 comes up (7 is the sum of the two die). Calculate $P(A|B)$ using the empirical approach.

4: Probability

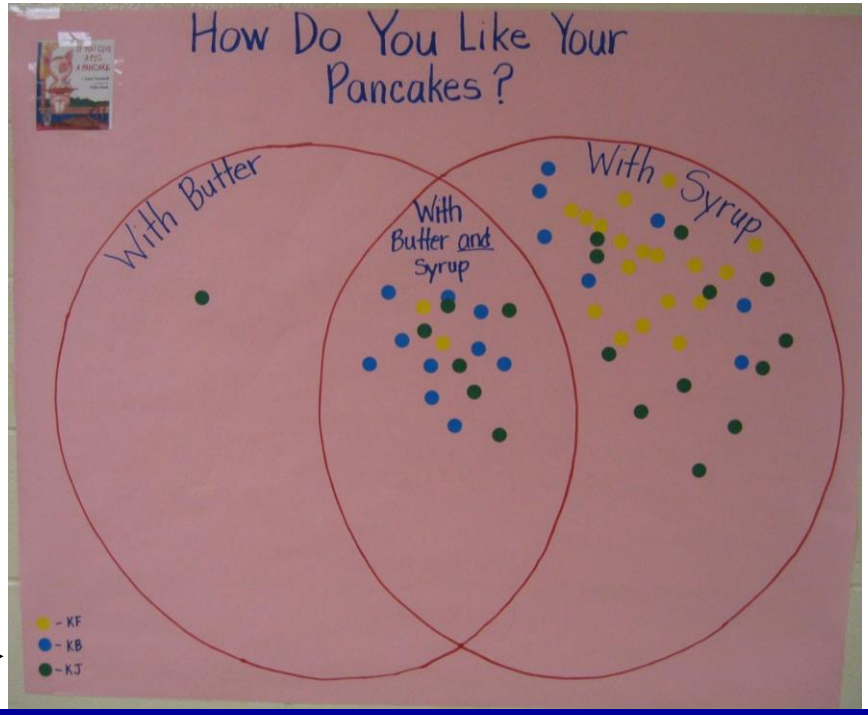
What is the probability that a random kindergartener likes syrup (B) on their pancakes given that they like butter (A) on their pancakes?

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)}$$

$$P(A) = \frac{n(A)}{n}$$

3 kindergarten classes →



4: Probability

Homework: Watch Catch 21 Episode
https://www.youtube.com/watch?v=N73GnjXfl48
The bonus round begins at time 12:28.
After the Q♠,J♣,A♠ are drawn:
What is the probability of an A as the 4th card?
What is the probability of a "10" as the 4th card?
What is the probability of an A as the 5th card?
What is the probability of an A as the 6th card?
What is the probability of a 6 as the 7th card?
(10 Card is a 10 or a face card.)